

THE EFFECT OF CERTAIN FACTORS IN
THE VERBAL ARITHMETIC PROBLEM
UPON CHILDREN'S SUCCESS IN
THE SOLUTION

A DISSERTATION
SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE
JOHNS HOPKINS UNIVERSITY IN CONFORMITY WITH
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
1930

BY
GRACE A. KRAMER

(Reprinted from "The Johns Hopkins University Studies
in Education," Number 20)

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TO MY MOTHER



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CHAPTER I

THE PROBLEM AND ITS BACKGROUND

This study was undertaken in the interest of improvement of the teaching of arithmetic. It seeks to investigate the effect of certain factors, and of our present trend of thinking and practice concerning them, upon children's success with the verbal arithmetic problem. That the school is eminently unsuccessful with the arithmetic problem is our common experience. In fact, there has developed within recent years a growing tendency to turn the emphasis in the classroom to drill upon the fundamental operations with integers and fractions with accuracy and speed as goals. Attention seems to be concentrating upon development of the psychology of mastery of the mechanics of arithmetic, rather than upon analysis of the types of reflective thinking essential for success with the verbal problem.

The arithmetic problem does not readily lend itself to critical study. It is difficult to isolate the factors which constitute the material and procedure of children's mental processes in problem solution. The steps in computation and the errors children make give evidence of the trail of their thinking in mechanical work, but we are not able to improve their arithmetic reasoning because we are not aware of their habitual ways of thinking in dealing with problems. If it were permissible, many teachers would probably turn their energy to securing "one hundred per cent accuracy in computation." There being still the course of study to be reckoned with, we have carried over the buoyant psychology of interest, meaningfulness, and colorfulness which has vitalized and socialized our work in content subjects to the field of arithmetic. In the development of this effort to make the work meaningful for children, to put meaning into school work as it is commonly expressed, we have wise guidance in such subjects as geography, history, and literature, but there has been little critical investigation or constructive thinking in the field of the arithmetic problem. It is a fact frequently stated by Dr. Judd of The University of Chicago that the

school is totally unaware of the extreme difficulty which arithmetic presents to children. We know almost nothing of the psychology of the arithmetic problem other than that children make unexpected and seemingly unreasonable reaction in the selection of principle, notwithstanding persistent practice. In an effort to lessen difficulty, and without much experimental guidance, we have tended towards adapting the psychology of interest to the arithmetic problem by seeking to put meaning into arithmetic, often by the use of problem situations external to arithmetic itself, such as were characterized by Dr. Thorndike¹ some years ago as "problems about Noah's Ark, or Easter Flowers, or the Merry-Go-Round, or a Trip down the Rhine."

The last few years have witnessed great activity in the production of arithmetic textbooks. Careful study of succeeding series impresses one that the conviction of these authors is that to make arithmetic interesting, to vitalize it, to socialize it is the obligation of the arithmetic textbook maker. A somewhat recent series, in fact, was scarcely to be distinguished from the typical school reader.² The question is not whether we are wise in turning from the old purely disciplinary problems, problems of the type described by Dr. Thorndike as "entirely devoid of any worthy interest, save the intellectual interest in solving them."³ We are seeking rather to clothe arithmetic with interest. The injunction of Dr. Thorndike is that we "try to find problems which will not only stimulate the pupil to reason, but also direct his reasoning in useful channels and reward it by results that are of real significance."⁴ One still finds in the modern textbook a number of problems clinging to the traditional materials of arithmetic, selling farms, building houses, finding the weight of potatoes, spending the month's salary, and so on. Certain textbook makers, however, employ widely the subject-matter of the school content subjects. Others employ the problems

¹ Thorndike, Edward L., *The Psychology of Arithmetic*, The Macmillan Co., 1922, p. 283.

² Buckingham-Osburn, *Searchlight Arithmetic*, Ginn and Co., 1927.

³ *The Psychology of Arithmetic*, p. 20.

⁴ *Ibid.*, p. 20.

of civic life, while many books make extensive use of the familiar activities of children, the pages of these books being given over in whole, or in part, to problems that have to do with parties, picnics, club activities, school and home projects. But arithmetic problem work does not improve commensurately with the care devoted to these materials and situations devised to insure interest. In 1929, in a certain great city school system, after an eighteen months' campaign for improvement in arithmetic results, the city-wide median success advanced in grade 6B from the first score of 32.2 per cent to a median score of 51.8 per cent. An administrator connected with this school system was heard to comment with placidity, "Maybe children learn only 50 per cent of the course of study in arithmetic." In this drive the emphasis was placed mainly upon the mechanics of arithmetic. Had the campaign been for improvement in problem solution, would it not have been extravagant to hope for better showing?

DEFINITION OF THE PROBLEM OF THIS STUDY

In the Strayer survey of 1921 Baltimore was found strong in arithmetic, but we are keenly alert to means of improvement in the teaching of this subject. Under the direction of Dr. Florence E. Bamberger, Professor of Education, The Johns Hopkins University, the study here reported was begun in the Baltimore schools in 1924. It has had the cooperation of Superintendent Weglein, of Dr. John L. Stenquist, director of Research, and of my colleagues on the staff of the Bureau of Research. The work commenced with a preliminary survey conducted with nearly one hundred 6B children in School No. 99 to study their reaction to a number of verbal problems selected from current printed sources. Careful analysis of these children's work indicated that lack of understanding of the meaning and significance of technical terms seriously handicapped them and that they were prone to multiply when in doubt as to the operation demanded.

After this exploratory study, the problem was narrowed to intensive study of the effect of variation in four factors selected as intrinsically inherent elements in the verbal arith-

metic problem. Each of these factors is a significant determinant of the nature or form of the arithmetic problem. The factors chosen were:

1. Interest
2. Sentence Form
3. Style—Details
4. Vocabulary

These specific factors were suggested both by the trend in textbooks previously mentioned and by the tendency to bend arithmetic to the policy of making school work meaningful and attractive for children. The investigation was begun with an open mind but with the hypothesis that success in problem work in arithmetic could not but be greatly magnified by providing interesting content and attractive problem form. As the study proceeded, however, it revealed a great deal concerning the mental processes which children actually use in problem work and much that should lend itself to improvement in our teaching.

By no means was it the plan to investigate thoroughly even one of the four factors selected. There had been little experimentation with the verbal problem upon which to build; most of the literature found at that time in treatment of the arithmetic problem seemed to be the opinion of the author, plausible and apparently reasonable inferences from experiences in other school subjects, but, after all, only scientific opinion. This study planned to attempt to control certain elements constituting these four factors and to vary them, one, two, three, and four at a time in the effort to secure definite measurements of success under known language and interest conditions.

THE INTEREST FACTOR

Interest as studied in this investigation is not of the superior type of the class venture actually in course of process such as, for instance, the estimating of the expenses of a class excursion, or plans for the purchase of a school radio to be used by the children doing the arithmetic. One readily concedes that the actuality of these conditions is not to be

equalled in devising problems for the arithmetic textbook. The concern is here for the "described problem," the vicarious experience, as Dr. Bobbitt of The University of Chicago terms it in discussing reading materials.

The newer arithmetic textbook usually expresses its author's conviction that the described problem should be made interesting to children, but one sometimes looks askance at the matter deemed intriguing to elementary school children. In a recent textbook there occurs the following problem:

It is said that Methusaleh lived 969 years. How long would it have taken him to haul $\frac{1}{2}$ ton of stone on an oxcart to a city 20 mi. away and to return, if his oxen went 2 mi. an hour and he traveled 10 hr. a day? Strayer-Upton Arithmetics, Middle Grades, 1928, p. 197.

In a textbook published in 1926 one finds the following problem, formulated evidently with an interest reference:

Abraham Lincoln received \$.03 apiece for splitting rails. How many rails would he have to split to earn an amount equal to the price of a ready-made suit today costing \$34.50? Knight, Ruch, Studebaker, Standard Service Arithmetics, Grade 5, 1927, p. 292.

These are both excellent textbooks, the work of widely known experts in arithmetic and it should be said that the examples cited are by no means typical of the content of the books. They are cited merely to illustrate the groping for material which in other connections would probably engage children's interests and which will, at the same time, lend itself to arithmetical computation. Certain authors devise long series of problems incident to an intriguing situation set up, as for instance the picnic situation with its series of eighteen consecutive problems presented in a modern textbook.⁵

An arithmetic published a year earlier⁶ devotes a number of chapters to discussions for the children of *Our Supply of Meat*, *A Story of Coal*, and similar problems of civic and economic importance, there being given with each topic groups of related arithmetic problems. Another new text, mentioned previously, employs series of problems under captions such as *Arithmetic in Geography*, *Population of Cities*, *Problems*

⁵ Buckingham-Osburn, Searchlight Arithmetics, Book III, 1927, p. 41.

⁶ McMurry and Benson, The Social Arithmetic, 1926, pp. 56, 163.

from *American History*, introduced, no doubt, because the authors are of the opinion that certain interesting history subject-matter will lend itself to manipulation as material for problem solving in arithmetic. Samples of problems from these sets are as follows:

1. At 4c. a mile how much would it cost for railroad fare from New York to San Francisco by way of Chicago? (Map furnished)
2. How many times as far is it from Chicago to New Orleans as it is from Boston to New York? (Map)
3. When Chicago was $\frac{1}{5}$ its size in 1920, how large was its population? (Scale)
4. In the United States in 1920 there were 27,661,880 men and 26,759,952 women of voting age. How many people of voting age were there in 1920 in our country? ⁷

The present investigation concentrates on a narrow angle of the various types of interesting material. A survey of textbooks in use at this time shows that arithmetic content now embraces four types of material used to secure interest:

1. Artificial motivation:

Here are some problems that hundreds of children who live in London, England, have done. Let us see how well American children do them when the problems use American instead of English money. The average London pupil ten years old can do the ten problems below.⁸

2. Material which in content or form seems to be extrinsic to arithmetic itself:

Mr. Smith has promised to sell 10 quarts of berries to his neighbor. He hired his children, John and Ida, to pick the berries. The children started picking, but they did not get along very well. John ate so many while he was picking that he had only $4\frac{3}{4}$ quarts left. Ida, who was quite little, became so tired after she had picked $2\frac{3}{4}$ quarts that she had to stop. How many more quarts must be picked if Mr. Smith is going to keep his promise? ⁹

3. Material adapted from content subjects, history, geography, and so on.

4. Material drawn from children's activities and evidently supposed by the authors to be of interest to them.

⁷ Knight, Studebaker, Ruch, *Standard Service Arithmetics*, Book III, pp. 203, 207, 255.

⁸ *Standard Service Series*, Book III, 1927, p. 91.

⁹ Buckingham-Osburn, *Searchlight Arithmetics*, Book III, 1927, p. 127.

This fourth type of supposedly interesting arithmetic material has been chosen as an experimental factor for this study. It is a type of content widely used in our textbooks and extensively pervading our classrooms. To use as arithmetic content material drawn from the children's own interests and activities is, in our modern parlance, to vitalize the work, to socialize it, to make it meaningful, to bring it to the child's own level. This study attempts to measure to what degree provision in this direction improves children's actual success in arithmetic in a given experimental situation.

The first task was to differentiate the problem to be classified as interesting with our present limited scientific knowledge of children's interests, from the problem which would fall outside this classification. An effort was made to gather scientific criteria of children's interests. Most often, these proved to be reading interests, there being little material available in other connections. There came opportunity to consult Dr. Lewis M. Terman in this regard in consequence of which the work of Mrs. Wyman, presented in the first volume of *Genetic Studies of Genius* (p. 455) was used as one basis of discrimination for interest. There is definite suggestion also in Dr. Fannie W. Dunn's investigation of reading interests in primary material¹⁰ and further practical criteria for guidance in Dr. A. M. Jordan's study of the interests of older children.¹¹ Dr. Wyman's tables furnish data for gifted and control children in scholastic interests, occupational interests, preferences for various types of activity, collections, and play interests, the percentages being given separately for both boys and girls at successive age levels. Use was made also of the reading interests study conducted by Dr. Terman and Margaret Lima, the outcomes of which are presented also in Chapter XIV of Terman's work.¹² That entire investigation, as is probably well known to the reader, was

¹⁰ Dunn, Fannie W., *Interest Factors in Primary Reading Material*, 1921.

¹¹ Jordan, Arthur M., *Children's Interests in Reading*, 1921.

¹² Terman, L. W. et al., *Genetic Studies of Genius—Mental and Physical Traits of a Thousand Gifted Children*, Vols. I and II, Stanford University Press, 1925.

undertaken by Dr. Terman with the cooperation of Dr. James C. DeVoss, Dr. Truman Lee Kelley, Dr. G. M. Ruch, Dr. Dorothy Yates and others to determine in what respects the gifted child differs from the typical child of normal mentality. "The aim," says Dr. Terman in the preface, "has been to collect, as far as possible, information of objective nature. . . . In the main, however, the conclusions are based upon well-defined experimental procedure which can be repeated *ad libitum* for purposes of verification or refutation." In the I.Q. distribution of the groups investigated (Vol. I, p. 47), one finds 905 unselected children with a range of I.Q. from about 65 to about 135, and a gifted group of 999 children with a range of I.Q. from about 135 to over 195, this latter group including probably 309 high school subjects who were studied in an extension of the work. This investigation and its outcomes are without question definitely reliable, and since the ranges of intelligence and of age cover, within reason, the ranges used in this study, its interest criteria have been deemed adaptable for use here. Tables have been used from the experimentation with *Scholastic, Occupational, and Other Interests; Play Interests, Knowledge, and Practice; Reading Interests; Tests of Intellectual, Social and Activity Interests* (Vol. I, pp. 363-482).

APPLICATION OF INTEREST CRITERIA TO ARITHMETIC TEST MATERIAL

In order to isolate and, in so far as possible, control the interest factor for purposes of measurement the following procedure was adopted. After a wide study of textbook material, a group of eighty-four problems was selected from textbooks actually in use in Baltimore classrooms together with others recently off the press. These problems were fairly representative of a mass of material accumulated for this phase of the work. The type problem here sought was the traditional compressed problem, in most cases there being no evident effort to engage children's interest in the situation presented in the problem. Interspersed in recent textbooks there are many such problems still in use, in earnest of the

lingering of the conviction that arithmetic may be in and of itself interesting to children. As the case may be, with the permission of the late Marshall Stitely, principal of School No. 22, and the cooperation of Miss Dellone, vice-principal, in sets of four tests this assemblage of, in great part, presumably uninteresting material was given a try-out in two 6B classes, this being the grade selected for the experiment. The entire group enrolled eighty-four children with a somewhat normal range of intelligence (I.Q. 72 to 142) and a wide spread in arithmetic ability, as shown by the Baltimore Course of Study test given a few months previously. The outcome of the try-out confirmed the experience of 1924. Children's problem work is disconcerting and disappointing. However, these were typical problems from reliable and accepted modern textbooks and the results served as foundation for the formulation of the experimental tests. From this group of problems, there were selected for final use a number of problems which did not meet the standards of interest as defined by available criteria for interest in the age range common in 6 B. These, with similar problems, were grouped as *Uninteresting*. This characterization of interest is relative rather than absolute. For individual pupils, certain of the problems sectioned as uninteresting may offer a certain degree of intrinsic interest, since interest is a function of one's experience and varies with that experience. The case is, rather, that because the situation involved in the problem is not one that involves a criterion definitely establishing interest, the problem is presumably uninteresting. The problems thus classified are of the traditional arithmetic type of content and each of them has been subjected to careful scrutiny to insure avoidance of the old purely disciplinary problem. Each is a real, genuine problem of the type common to the textbook of the last decade, the situation presented having been almost invariably selected, adapted, or patterned from textbooks now in use or contending for adoption in the schools.

It was necessary to formulate the interesting problems especially for the investigation. Careful study was made of

the problems offered as presumably interesting in various textbooks. The aim was to avoid extreme or fantastic problems and to secure test material similar in style and theme to that in use in our classrooms. The interests and activities selected for the research material conform rather definitely to available scientific criteria of children's interests. The location of these actually interesting activities was followed by consultation with school people working in the classrooms in order that the situations devised for the problems might be definitely tied up with actual experiences of school children as they occur in Baltimore. The Public Athletic League furnished figures reasonable for competitive games and Miss Engle of the Home Economics Department furnished data used in problems concerning certain practical activities.

The interest in the problem, it may be well to note, does not attach itself to the mere stimulus word, as one might conceive it to function in an association test. No proof could be adduced that interest such as is generated by a stimulus word might not attend the reading of our least interesting problem. Rather it was sought to devise engaging problem situations involving the interest in question, problems in which the situation itself would intrigue the attention of children of the age range and grade involved. No problem has been given place as interesting which cannot by one or more references be shown to employ an experimentally determined criterion of interest. It was the plan to select, in most instances, interest situations common to both boys and girls. In the tests, however, there occur, as one would reasonably expect, a few problems built on interests more characteristic of boys, as in like manner a limited number with high percentage of interest for girls. Special accounting is rendered for such problems.

At the close of this study, question arose as to whether reading and activity interests are potent in arithmetic. To support the *a priori* judgments with experimental evidence, a representative block of the test material was submitted to children's choices. An agreement of 83 per cent was found between the children's expressed preferences and the origi-

nal classification for interest. This second study is described at the close of the major report.

The interesting problem involves an activity interesting *as an activity* to children of this age or, if it be a reading interest, a situation about which they like to read, the purpose being to measure in how far the interesting problem surpasses the uninteresting problem as an impetus to right arithmetical thinking. Will boys and girls succeed better with problems about their own activities and interests than they do with the traditional problems of arithmetic? Will the interest in the situation *per se* produce similarly keen interest in the arithmetical problem involved, interest such as would be manifested by care in thinking through the solution? Or is it that children's mind sets in their interesting activities and enjoyable reading are so different from the mind set in arithmetic that the one does not furnish drive for the other? Does improvement in arithmetic attend our plan of projecting arithmetic into the realm of children's characteristic activities and interests? We shall see.

SENTENCE FORM

The second factor set up for study in this investigation is the influence of the sentence form in which the problem appears upon children's success in selecting the correct principle of solution. One half of the problems are of the proverbial, one might almost say invariable, type of the textbook of five years ago, the form in which the facts and the requirement are all given within the confines of a single complex-interrogative sentence. This form is still in use in textbooks and, since it is the purpose here to study trends of present usage, most of these problems are introduced by the subjunctive "if" clause. To provide variety, however, and to keep the test material as near to the modern textbook material as possible, conditional clauses in "when" have been occasionally employed, as also heavy nominative absolute participial phrases introduced by the customary textbook "allowing,"—

Allowing $5/8$ yd. of material to a vest for a man's suit of clothes, how much,¹³ etc.

The participial phrase in these instances has the character and function of the conditional clause. The material is somewhat more fully representative in that it includes also some problems of the "buried if" type.

The contrasting problem form is of the type which for convenience we have designated "Declarative," the form now gaining place in textbook usage. The problem, in this case, is introduced by a declarative sentence, factual material incident to the problem being given through the medium of the complex or compound declarative sentence, or in some instances, by two or more declarative sentences. These problems employ the ordinary form and style of factual material, the question being asked by a distinct interrogative sentence or by an imperative sentence in "Find": *Find the total yield of 27 acres.* With these two sentence forms the investigation seeks to compare success in two contrasting problem forms. The one is a form of discourse encountered nowhere else than in the mathematical problem. There is given a condition, a suspension, with factual material and question bound together in one sentence, a purely problematic form of language. The declarative form follows the tendency manifested in recent textbooks to use ordinary discourse or story form. With a sudden jerking up, the child's attention is directed to the actual significance of what has been presented by a question demanding arithmetical response. By an easy step forward textbook makers have concluded that arithmetic would profit by adoption of the style found pleasing to children in ordinary reading material. There is no experimental evidence that the declarative form is more effective than the old suspended form. They have merely followed what seemed a reasonable presumption that it might be, and have expected forthwith better results from children. The data from this investigation were in hand and under statistical interpretation when Dr. Buck-

¹³ Smith-Burdge Arithmetics, Intermediate Book, Ginn and Co., 1926, p. 115.

ingham of Harvard University, in a conference with administrators in Baltimore, noted the need for definite knowledge concerning the influence of the sentence form of the arithmetic problem.

DETAILS

The third phase of the investigation studies the effect of the use of details in setting forth the problem situation. One half of the problems are briefly stated without details; in the other half the details of ordinary discourse are employed. The purpose is to compare the effect of the traditional compression of the arithmetic problem with the effect of adopting the style usually employed with factual material. The details are in nowise devised to clog thought, nor are they extraneous to the situation. They are merely the details inherent in the situation in its natural setting. They are, moreover, language details, no unneeded numbers being introduced in any of the problems.

The interesting problems without details rely for interest upon the skeleton structure of the situation. Where details are in order, the problem approaches the story form. In no case has it been the purpose to employ fantastic details or to contrive unusually cumbersome forms, since such is scarcely the present usage in any textbook on the market. Although in many of the condensed problems not a word could be dropped, several of them employing only about a dozen words, no problem is so condensed as to be elliptical. In both instances, it has been the aim to compare the two trends found flourishing together in textbooks,—the compressed, matter-of-fact traditional problem and the problem employing consistent factual details intrinsic and inherent to the situation described, the former being the style native to arithmetic, the latter the usual style of well-written material. The uninteresting problems in many cases are either quoted exactly from textbooks now flourishing on the market or with such adjustments as were essential to comply with the obligations imposed by the factors under study, the permission of textbook publishers having been duly obtained for use of their materials.

VOCABULARY RESEARCH WORK TO ESTABLISH CRITERIA

The fourth factor selected for investigation in this study is the effect of unfamiliar vocabulary upon children's success in problem solution. Little is scientifically or reliably known of actual vocabulary or word difficulty for children beyond the level of the primary grades. The comprehensive study reported in Dr. Thorndike's *Word Book* presents "a list of the 10,000 words which are found to occur most widely in a count of about 625,000 words from literature for children; about 3,000,000 words from the Bible and English classics; about 50,000 words from books about cooking, sewing, farming, the trades, and the like; about 90,000 words from the daily newspapers; and about 500,000 words from correspondence. Forty-one different sources were used."¹⁴

Dr. Thorndike explains that his investigation seeks to ascertain answers to the questions, "How widely is the word used? How often is the word used?" In his article introducing the *Word Book* to educators (*Teachers College Record*, September 1921) it is evident that Dr. Thorndike had in mind, not word difficulty as one might ordinarily conceive it, but rather word importance as indicated by relative word frequencies. He leaves no doubt in the mind of the reader that he planned the list to serve as a reliable and immediately useful body of material for the teacher's guidance in dealing with matters of reading vocabulary and in evaluating the grade appropriateness of textbooks from the vocabulary standpoint. The work is of monumental significance in our language although after a period of ten years of development in English usage, many words would, by revised count, now give different frequencies. This, however, was its author's expressed expectation. The criticism of certain vocabulary students that the *Word Book* does not adequately serve as a sole measure of word knowledge for children is valid but that its author planned some such reliance is evident from various passages in the introductory article mentioned:

Conscientious teachers now spend much time and thought in deciding what pedagogical treatment to use in the case of words which offer

¹⁴ Thorndike, Edward L., *The Teacher's Word Book*, 1921, p. 3.

difficulty to pupils. In the third readers which they use they find, according to Miller, over nine thousand different words. Some of these probably should not be taught at all in that grade. . . . Some should be taught thoroughly and reviewed. *The Teacher's Word Book* helps the teacher to decide quickly which treatment is appropriate. . . . The same service is performed, of course, in each of the school grades.¹⁵

One element in such an evaluation of almost all textbooks is the suitability of their vocabulary to the grade for which they are intended. This can be measured with absolute impartiality with the aid of the list.¹⁶

Just as word counts of such material as the pupil may need to write are instructive in the pedagogy of spelling, so word counts of such material as the pupil may need to understand will be instructive in the pedagogy of reading, and indeed in all of the school subjects which are presented with the aid of language. So for about ten years I have made such counts as I could.¹⁷

Dr. Thorndike, however, makes no definite grade allocation for words, although in his recent publication of the outcomes of his investigations in adult learning¹⁸ he ventures the statement that "the average pupil in grade eight knows after a fashion the meanings of about ten thousand English words." Nevertheless, one finds scientific investigators using the Thorndike list as a criterion of word difficulty in children's material if one assumes difficulty to mean likelihood of the word's being unfamiliar to pupils of the grade in question,—the new, the unfamiliar being probably accepted by them as synonymous with difficult. Certain words, on the other hand, present inherent difficulty to children as units of recognition because of unusual or peculiar configuration; "weigh," and for the first grade child "was" are examples. In the writer's previous investigation, conducted among children who had experienced trouble in learning to read, the learning process was found to be impeded by specific words ordinarily assumed to be easy for even the beginner.¹⁹ It is not difficulty in the sense of configuration that this study has in mind. From this viewpoint,

¹⁵ Thorndike, Edward L., *Word Knowledge in the Elementary School*, Teachers College Record, September 1921, p. 355.

¹⁶ *Ibid.*, p. 359.

¹⁷ *Ibid.*, p. 334.

¹⁸ Thorndike, Edward L., *Adult Learning*, The Macmillan Company, 1928, p. 149.

¹⁹ Kramer, Grace A., *Reading Failure in Third Grade*, M. A. Thesis, The Johns Hopkins University, 1924, unpublished.

there would have been no criterion available to guide the selection of vocabulary or to serve as a measuring rod to differentiate at any intermediate grade level the comparative familiarity or unfamiliarity of words.

The Thorndike *Word Book* has been accepted in this study as a basis for determining word familiarity. Since it was necessary to drop a plumb line at some definite level to provide a working basis, words below the level of 4b (3500) were deemed familiar and words appearing at 4b or beyond, for the purpose of this investigation, relatively unfamiliar. It was a plumb line of the relative value of words in grade 6 that the investigation had in mind rather than of inherent word difficulty. The lower range covers the first 3500 words of the 10,000 words in the *Word Book*. There is implication of Dr. Thorndike's thinking in his having provided, as a supplement to the *Word Book*, a list of "the 2500 words of most wide and frequent occurrence."

That Dr. Thorndike's list is accepted, from some points of view, as a criterion of reading difficulty by reputable investigations is established by one's frequently coming upon it in scientific studies. Dr. Terman writes, in introducing the investigation of intellectual, social, and activity interests, that the task "of selecting a valid method" was entrusted to Mrs. Wyman. In her free association test,²⁰ in which the word was presented visually, that is as a reading stimulus, and at the same time pronounced, one criterion for word selection was that the word appear in the first 2000 of Thorndike's Word List, the preliminary try-out having been made in grades 6 and 7 although the final testing covered a range of grades 4 to 8. The highly specialized purpose for which these stimulus words served exerted, no doubt, major influence in their critical selection.

Dr. Thorndike's list has probably had its most recent scientific application in the building of the Word Meaning Test for the New Stanford Achievement Test. "The Thorndike *Word Book*," the authors explain, "was used to determine the difficulties of both the critical and response items.

²⁰ Terman, L. W. et al., *Genetic Studies of Genius*, Vol. I, p. 457.

The critical words have in nearly all cases a value of 2000 to 3000 less than the response words. The range of frequency extends from words in the first hundred to words between the ninth and tenth thousands in difficulty. In order to extend the list in the upper ranges, a number of words judged to be more difficult than any of those in the Thorndike *Word Book* were added.”²¹

Not all words in any area of frequency lend themselves to the arithmetic problem. A wide range of adverbs and adjectives, for instance, proves impracticable but, in so far as the nature of the arithmetic problem permits, this study attempted to escape the bondage of the usual arithmetic vocabulary and to use a reasonably wide range of vocabulary. As English is constructed, however, a great number of simple, common words, more than one might ordinarily suspect, occur in even our best stylists, these words being the sinews, as it were, of discourse.

When the first interpretation of results of the effects of familiarity and unfamiliarity of vocabulary caused question to arise as to whether word familiarity could be determined solely upon position in the *Word Book*, further criteria were sought for evaluation of the vocabulary phase of the problem.

The research test vocabulary was in consequence scaled on the Gates *Primary Reading List*, it being the assumption that words appearing therein might be deemed familiar and useful in Grade 6 B. This list,²² as the reader will recall, uses five criteria of utility, three of interest and two of difficulty, the difficulty criteria being:

1. On assumption that words commonly used in speech will be easier to learn to read than those less commonly used
2. As determined by length and configuration of words

The frequency of the appearance of the word in children's spoken language served to give numerical rating for

²¹ New Stanford Achievement Test: Guide for Interpreting, World Book Company, 1929, p. 5.

²² Gates, Arthur I., *A Reading Vocabulary for Primary Grades* (Introduction), Teachers College Bureau of Publications, Columbia University, 1926.

the first criterion but the second could not be measured by quantitative device. This work of Dr. Gates is our most reliable and widely used primary reading list and here, too, Dr. Thorndike's previous work has been accepted as providing points of cleavage. This list, then, which provides 1500 words suitable for use in primary reading material served to establish, by a second criterion, the relative position of words to be rated familiar, or rather, not unfamiliar, in the present study.

The vocabulary was then checked on the Pressey lists for arithmetic and general mathematics.²³ These subject vocabulary lists attempt to establish those words which should be taught for intelligent pursuance of the subject in question without reference to the relative or intrinsic ease or difficulty of the words themselves. They note words incident to textbooks, words conveying arithmetical ideas and concepts, and so on. The lists have served to indicate in our case the occurrence of the technical words of arithmetic in the experimental material.

At this juncture the aid of certain investigators in the field of children's vocabulary was sought. Probably Dr. Arthur Gates of Columbia University will permit reference to the statement that he deemed the Thorndike list a satisfactory criterion of usefulness of words above third grade and that in his own studies of the difficulty of recognition and pronunciation of words, he had found these forms of difficulty to be substantially correlated with the Thorndike ratings. His objections, he writes, to the Thorndike material for the primary period do not hold in the case of the higher grades. Dr. Gates generously permitted the use in our investigation of the manuscript of "the next few hundred words" beyond the 1500 words in his *Primary Reading List*. In checking against this supplementary material, as also with the 1500 word list, one presumably further established the range of familiarity of vocabulary, the hypothesis being that words which are not definitely known to be common, useful, and familiar have the presumption of a varying degree of un-

²³ Pressey, L. C., *The Determination of the Technical Vocabulary of School Subjects*, School and Society, XX (1924), 91.

familiarity. It would seem from this study that familiarity of vocabulary is somewhat definitely established as is a possible beginning range of unfamiliarity. Beyond this, one faces a dearth of scientific evidence. It is here that a most valuable contribution might be made to our present knowledge of relative word usefulness for older children.

Since Dr. Ernest Horn, of the University of Iowa, had made wide vocabulary studies in the compilation of his 10,000 words most commonly used in writing,²⁴ his assistance was sought also in the search for further vocabulary criteria. Dr. Horn comments that we need badly data regarding the reading difficulty of various words. It is his opinion that at the present the best available data to use for that purpose seem to be counts which have been made of the words which children most often use in talking and writing, and that it would be fair to assume that words frequently used by children in speaking or writing may be counted upon with assurance as being easy to read. Dr. Horn stated further that he had in his files the tabulated summary of all of the investigations of speaking and reading vocabularies in so far as he had been able to learn of such investigations and that he knew of no better way to measure reading vocabularies than to check them against these summary tables. It is the good fortune of this study that the vocabulary of the experimental arithmetic tests was checked at the University of Iowa against Dr. Horn's summary lists. The various sources of the ten different word counts entering into this measurement of the research test vocabulary are as follows, several of them being widely known and others not readily available:

1. Occurrence in Mrs. Horn's study of the vocabulary of kindergarten children²⁵
2. Occurrence in Mrs. Horn's vocabulary study of children of kindergarten age (Picture stimulation)²⁶

²⁴ Horn, Ernest, *A Basic Writing Vocabulary*, University of Iowa Monograph, 1926.

²⁵ *A Study of the Vocabulary of Children before Entering the First Grade*, Bulletin of the Child Study Committee of the International Kindergarten Union, 1926.

²⁶ Unpublished.

3. The use of the word by children at home (five years old or less)²⁷
4. Grade occurrence in Jones's concrete investigation of the material of English spelling²⁸
5. Grade occurrence in the New Orleans list²⁹
6. Grade occurrence in Tidyman's survey of Connecticut children's writing vocabularies³⁰
7. The earliest grade in which the word was found to be misspelled by Cooper in his study of words misspelled by Minneapolis children³¹
8. Occurrence in Packer's study of first readers³²
9. Occurrence in Housh's study of second readers³³
10. Occurrence in Miller's study of third readers³⁴

As has been noted with previous data, the criteria available through Dr. Horn's courtesy tends also to establish the range of familiarity of vocabulary. The case must rest, it would seem, until we have further data, upon the assumption that words that have not by investigation been proved familiar, have reasonable presumption of unfamiliarity.

Dr. Francis D. Curtis of the University of Michigan has also made studies in vocabulary. Although he could suggest no definite measuring rod, his thinking throws further light upon the status of our present means for determining word familiarity. Probably one might within courtesy quote him directly:

"It is my opinion," he writes, "based upon no objective evidence whatever but upon rather extensive observation, that Dr. Thorndike's list is away over the heads of seventh and eighth grade pupils as I know them. I feel as reasonably

²⁷ Unpublished.

²⁸ Jones, W. F., *Concrete Investigation of the Material of English Spelling*, University of South Dakota, 1915.

²⁹ New Orleans Public School Spelling List, Hansell and Brothers, New Orleans, 1916.

³⁰ Tidyman, W. F., *Survey of the Writing Vocabularies of Public School Children in Connecticut*, Leaflet No. 15, Bureau of Education, Washington, D. C., 1921.

³¹ Cooper, Edwin J., *A Spelling List of Pupils' Misspellings*, M.A. Thesis, University of Minnesota, 1927.

³² Packer, J. L., *The Vocabularies of Ten First Readers*, edited by Dr. Horn and published after author's death, *Twentieth Yearbook of the National Society for the Study of Education*, 1921.

³³ Housh, E. T., *Critical Analysis of Ten Second Readers*, M.A. Thesis, University of Iowa, 1916, reported in *Seventeenth Yearbook of the National Society for the Study of Education*, 1918.

³⁴ Miller, W. G., *Critical Analysis of the Vocabularies of Ten Third Year Readers*, M.A. Thesis, University of Iowa, 1916.

certain as one can, lacking objective data, that there are many words in the upper five thousand of Thorndike's list which would not be familiar to many junior high school pupils. I should personally prefer, therefore, to limit my list to the lower five thousand rather than to the ten thousand, at least for the seventh and eighth grades."

It was an hypothesis of this nature which led to the setting up of the 4B level of difficulty as the arbitrary line of cleavage for this investigation.

After the test vocabulary had been checked on Dr. Gates's printed and extended lists, by Dr. Horn's summary tables, by the arithmetic vocabulary test formulated by Sara E. Chase³⁵ and by the common mathematics and arithmetic sections of Pressey's *Technical Vocabularies of the Public School Subjects*, there came to hand the recent study of Dr. Galter³⁶ dealing with the improvement of the spoken vocabulary of elementary school children, an investigation covering grades one to six, inclusive. The monograph offers a tabulation of words and phrases submitted by Philadelphia teachers as having been found useful in their respective grades during a semester. Dr. Galter has by statistical treatment arranged the 2669 words in grade lists. It is somewhat pertinent to the present investigation that he has listed the Thorndike placement of the words and, in certain instances, his own grade re-allocation:

"It is assumed,"³⁷ says Dr. Galter, "that other words in the Thorndike list not suggested by the Philadelphia teachers are equal in difficulty with the words suggested. We can generalize, therefore, by saying that the words falling in Thorndike's first 500 are known to 75 per cent of the children in grades I, II, and III and the teachers of these grades may safely omit these words from special drill. In the same way are allocations made for each grade and for each quartile in each grade. While it is possible to apply this classifi-

³⁵ Chase, Sara E., *Waste in Arithmetic*, Teachers College Record, XVIII (1917).

³⁶ Galter, Israel, *Improving the Spoken Vocabulary of School Children*, 1928.

³⁷ *Ibid.*, p. 25.

cation to all the words of the Thorndike list, it is primarily intended as a basis for classifying the words selected in this study." Dr. Galter's final table for the grade allocation of words (p. 30) runs as follows:

FINAL ALLOCATION OF WORDS FOR ACTIVE CHILD VOCABULARY
(Words Selected on Basis of Thorndike Frequency and on Quartile
Distribution of Table VII)

	Grade I	Grade II	Grade III
A*	Less than 500	Less than 500	Less than 500
B**	500-2000	500-2500	500-3000
C***	Above 2000	Above 2500	Above 3000
	Grade IV	Grade V	Grade VI
A*	Less than 1000	Less than 1500	Less than 1500
B**	1000-4000	1500-5545	1500-6618
C***	Above 4000	Above 5545	Above 6618

* Too easy; known to 75 per cent of class

** Satisfactory for middle 50 per cent of class

*** Too difficult for 75 per cent of class

Figures represent frequency on Thorndike Word List

Inspection of the specific words deemed too difficult in the Philadelphia lists leads one to suspect that children's actual experiences are the potent factors in determining word difficulty. Certain of the words found actually too difficult for children forming that investigation group would probably not be deemed difficult or uncommon in the speaking vocabulary of sixth grade children in Baltimore, as for instance, "crusade." Sixth grade history deals with the Crusades, and, as history is taught in our schools, the child would actually for weeks live in the atmosphere of the Crusades in his classroom. The demarcation of a Thorndike frequency of 6618 (credit number of 7) as determining a level of cleavage for words too difficult for 75 per cent of the pupils in grade 6 might seem only somewhat in accord with the experiences of other vocabulary investigations, and then provided only that we note that the characterization is "too difficult" rather than merely "difficult." To students of vocabulary it may seem that the study places the outpost for active or spoken word vocabulary beyond a probable area of reading difficulty. With this reservation in

mind, the experimental test vocabulary was scanned for words which Dr. Galter's data would intimate to be too difficult for the spoken vocabulary of Grade 6, such words being indicated by an asterisk in the measurements of the test vocabulary given in Appendix II of the original report of this study on file and available in the library of The Johns Hopkins University.

By the cumulative checking against all these criteria, which are in certain instances inclusive of earlier studies, it is probable that the research arithmetic vocabulary has been subjected to as thorough measurement as one could reasonably hope to secure at this writing.

DICTION ANALYSIS CHART

Used in connection with the 15 vocabulary criteria to arrange the research test problems in ascending order of vocabulary unfamiliarity in the attempt to secure a more suitable measure for Grade 6B than is offered by the Thorndike *Word Book*.

Difficulty Index
No. of the Problem

All in Gates 1500
All in Gates extra 300
All in Gates except names
Thorndike upper level where consistent
Thorndike level disputed words
Horn level same disputed words
Not in Thorndike list
Abbreviations of units of measure
Thorndike level
Horn level
Chase study—per cent missed—Grades 4 and 5
Pressey—Arithmetic or general mathematics technical terms
Mathematical expressions not mere units of measure
Operation cues
Adult expressions—usage or diction
* Adult construction (Consequent on problem patterns)
Interpolation
Nominative absolute
Pronounced suspension

* Not permitted to influence position in vocabulary familiarity series, although the two proceeded with necessary relation.

An analysis chart, shown on the previous page, was then devised to guide in arranging all of the problems in language sequence from familiar to unfamiliar. The chart served as a diction measurement since both the style and diction of the problems were, in great part, determined by the choice of vocabulary. Beginning with problems so simple that all of the words occur in Thorndike's first 500 words, the sequence moves through levels of increasing word unfamiliarity. Concurrent with this mounting of vocabulary unfamiliarity, the analysis notes the occurrence of abbreviations of units of measure deemed as words rather than as symbols (bu. for bushel) ; the employment of technical, mathematical expressions such as do not occur in ordinary discourse ; familiar words used in uncommon commercial or adult sense ; and expressions to be characterized as adult idiomatic usage. All of these phases of diction, which are in themselves sensible increments of language difficulty, serve to bring about change in the problems from a simple narrative style to a comparatively difficult technical or adult style.

Tables are given for comparative success with familiar and unfamiliar vocabulary by the two separate standards. Percentages of success have been computed for the two vocabulary sections when assembled by the original plan, that is, with the first 3500 words in Thorndike reckoned as familiar and words above this limit, including a few which do not appear in the *Word Book* at all, as unfamiliar. It proved possible to classify the problems on this framework with but a few words out of position. A second comparison is offered with the two vocabulary sections determined by the composite analysis of vocabulary described. The problems were arranged in *seriatim* ascending steps of vocabulary or diction unfamiliarity by the fifteen vocabulary criteria³⁸ in conjunction with the chart shown on page 23. This second criterion is merely an attempt to combine available material into a supplementary vocabulary measure. The Thorndike frequency scale has been used as the stable criterion.

³⁸ See Appendix II of the copy of this study on file at The Johns Hopkins University.

CHAPTER II

BRIEF HISTORICAL SURVEY OF INVESTIGATIONS IN PROBLEM SOLVING IN ARITHMETIC

The history of arithmetic as a curriculum subject is a fascinating chapter in the genetic development of education in America. From the inconspicuous position which it occupied, certainly until the time of Pestalozzi, it rose to the position of the most important study in the school, the touchstone, as it were, for intelligence before the days of the intelligence test. In 1821 Colburn's arithmetic replaced the European cipher book in which pupils were wont to solve mechanically problems classified as definite cases under the rule, the motive being to eliminate mental work in the solution and to confine arithmetic to computation. For the next fifty years the new text, called *The Intellectual Arithmetic*, exerted influence in systematizing arithmetic and in improving teaching, the philosophy of its author being that arithmetic is an important discipline of the mind as well as a subject of practical utility.

Formalism, the Nemesis of the school, then again encased arithmetic in a bond of rigid drill and we have the static period which persisted until about thirty years ago when the general interest in reform in education spread to arithmetic. Although its psychology is perhaps the least understood of any of the school subjects, arithmetic has been peculiarly sensitive to changing philosophies of education. The work of G. Stanley Hall and the practical philosophy of Dewey turned a ray of light upon formal work in arithmetic. The earliest investigation came, perhaps, with the survey work of the Committee of Ten (1893) and that of the Committee of Fifteen (1895), both of which concerned themselves with lessening the emphasis upon arithmetic in the elementary school. In his pioneer testing, Rice¹ (1902) dealt specifically with the relation between time spent on arithmetic and the results obtained. It probably seemed in-

¹ Rice, J. M., *Forum*, 34 (1902), 281.

credible to schoolmen of his time that greater expenditure of time was not rewarded by greater achievement. Stone's work ² (1908) centered in comparison of the product of the first six years of work in arithmetic through study of the test results in 26 school systems, his findings being tersely expressed by *diversity, chaos, waste*. In 1909 Courtis ³ contributed the first of the type of studies which aims for the scientific standardization of arithmetic. Bonser's ⁴ study (1910) led to the conclusion that native ability is an important factor in arithmetic reasoning.

Notwithstanding these early objective beginnings, there persists in the arithmetic investigation a tendency to become discursive rather than exact, scientific, and quantitative. Because of the nature of the factors involved in the reasoning process, arithmetic does not readily lend itself to experimental determination. We have, therefore, applied to it certain of the analyses obtained in reading studies, with the expectation that the arithmetic product would show improvement commensurate with the phenomenal improvement in children's development of reading ability under guidance of known psychological principles. But arithmetic reasoning has not improved; arithmetic is still the major cause of failure and non-promotion in the elementary school.⁵ In those arithmetic investigations which use scientific method in accumulation of data, there is a persistent tendency to present conclusions through the medium of somewhat general suggestions, rather than by means of statistical interpretation. Students of arithmetic have a reliable compendium and evaluation of scientific work in the *Summary of Educational Investigations Relating to Arithmetic* ⁶ published by Profes-

² Stone, Cliff. W., *Arithmetic Abilities and Some Factors Determining Them*, Columbia University Contributions to Education, No. 19, 1908, p. 90.

³ Courtis, S. A., *Measurement of Growth and Efficiency in Arithmetic*, Elementary School Teacher, X, 58.

⁴ Bonser, F. G., *The Reasoning Ability of Children of Fourth, Fifth and Sixth Grades*, Columbia University Contributions to Education, No. 37, 1910.

⁵ Judd, Charles H., *Psychological Analysis of the Fundamentals of Arithmetic*, University of Chicago Press, 1926, p. 5.

⁶ Judd, Chas. H. and Buswell, G. T., *Summary of Educational Inves-*

sors Judd and Buswell in 1925 and supplemented annually by annotated bibliographies published in the *Elementary School Journal* by *The University of Chicago Press*. Their work confirms one's own experience that the emphasis of investigation has shifted to the computational phase of arithmetic not because we do not recognize the intrinsic value of reasoning, but because critical thinking in arithmetic apparently eludes quantitative study. Of the 532 arithmetic investigations reported in the summary monograph, certainly less than 10 per cent present quantitative data on the matter of problem solving.

It is the purpose of this chapter to note important scientifically controlled studies together with certain scientific inquiries significant in their conclusions. As was the case in reading, the testing movement awakened and encouraged interest in the verbal arithmetic problem. Monroe's work (1918) in the derivation of tests⁷ builds upon the earnest work of the men who preceded him, the major interest of his work being in the direction of the construction of adequate tests in arithmetic reasoning. He offers, therefore, critical analysis of certain factors in the arithmetic problem rather than a statistical study. With the realization that we have no satisfactory analysis of either the mental processes involved in arithmetical reasoning or of the subject matter of the problems themselves, he insisted, although the succeeding decade paid little heed, that it is pertinent to inquire what is the nature of the mental processes that occur in solving arithmetic problems and what are their essential characteristics or dimensions. His sensitivity to technical words as important data in problem solution is significant. The succeeding decade has recognized as technical those words which define quantities, but Monroe's wider inclusion of words which define quantitative relationships is overlooked to great extent even in Pressey's survey. Those contributing abilities and controls which constitute ability to

tigations Relating to Arithmetic, Supplementary Educational Monograph No. 27, University of Chicago, 1925.

⁷ Monroe, Walter S., *The Derivation of Reasoning Tests in Arithmetic*, School and Society, VIII (1918), 296.

solve verbal problems, Monroe deemed it not possible to isolate. He would, therefore, "give appropriate recognition to each through the constructing of a composite reasoning test." We have not proceeded far in isolating or defining these abilities and most investigators would admit that their data would lead them to concur with the opinion of Monroe that, "In many cases the pupil does not reflect . . . He automatically identifies the problem as requiring a particular operation." Curtis⁸ (1913) and Wilson⁹ (1917) conducted surveys to determine the arithmetic employed in ordinary life activities. In this early work in Indiana and Iowa, Wilson found that the problems reported by business men and other citizens as actually occurring in every day life are brief and simple. From his later study of social and business usage of arithmetic¹⁰ (1926) he concludes that usage is much simpler than present school practice. He proposes that the curriculum maker use as criterion the question, "Are the business demands of this community such as to require that this process be taught in the schools?"

This movement towards limitation of arithmetic is exerting wide and probably ill-guarded influence. Its strongest opponent is Dr. Judd, who on the platform and in his writings insists that to so limit arithmetic is to rob children of their heritage:

"Curriculum makers should be urged," he writes, "to recognize the fact that the curriculum is made for the purpose of training minds, not for the purpose of reflecting the immediate needs of practical living. There are psychological requirements which must be met in curriculum making, and these are infinitely more important for the future of the race than are the practical adjustments of trade and industry."¹¹

⁸ Curtis, S. A., *The Curtis Tests in Arithmetic*, Report to New York Board, 1913.

⁹ Wilson, G. M., *A Survey of the Social and Business Usage of Arithmetic*, Sixteenth Yearbook of the National Society for the Study of Education, 1917 (See also *Columbia University Contributions to Education*, No. 100, 1919).

¹⁰ Wilson, G. M., *What Arithmetic Shall We Teach?* Houghton Mifflin Co., 1926.

¹¹ Judd, C. H., *Psychological Analysis of the Fundamentals of Arithmetic*, 1927, p. 116.

In 1922, Dr. E. L. Thorndike published a general dynamic psychology of arithmetic, the culmination of fifteen years of experimentation and critical analysis, the experimental data being largely from the field of computation rather than of reasoning. In discussion of the verbal problem the book offers the expert opinion of its author and projects into the teaching of arithmetic his insight into the principles of educational psychology and mental growth. It is a case of the opinion of the master transcending in value the semi-scientific finding. Dr. Thorndike makes unrelenting protest against artificial problems, unreasonable vocabulary, and unreal situations as matters of dynamic psychology rather than as factors positively influencing success in arithmetic. The thinking of the genius here falls in a different category from that of the textbook-maker who upon mere inference or upon suggestion of the possible influence of certain factors upon the arithmetic product itself changes the content and form of the textbook and predicts consequent improvement in children's achievement. "It is not at all certain," says Dr. Thorndike, "that children in grade 2 get more enjoyment or ability from adding the cost of purchases for Christmas or Fourth of July, or multiplying the number of cakes each child is to have at a party by the number of children who are to be there, than from adding gravestones or multiplying the number of hairs on bald-headed men. When, however, there is nothing gained by substituting remote facts for those of familiar concern to children, the safe policy is surely to favor the latter."¹² In discussing the meaning of problem-attitude as a condition of learning, Dr. Thorndike says further: "It is bad policy to rely exclusively on the purely intellectualistic problems . . . It would be still worse policy to rely exclusively on problems arising outside of arithmetic. To learn arithmetic is itself a series of problems of intrinsic interest to healthy-minded children."¹³ These are expert opinions of inestimable value but, speaking with his characteristic caution, Dr. Thorndike reminds his reader that "the

¹² Thorndike, E. L., *The Psychology of Arithmetic*, 1922, p. 222.

¹³ *Ibid.*, p. 276.

scientific measurement of the abilities and achievements concerned with applied arithmetic or problem solving is a matter for the future."¹⁴

A practical type of investigation of factors influencing the verbal problem has been carried on in connection with study and revision of the arithmetic textbook. In his examination of eight textbooks for beginners, Thorndike¹⁵ found that slight attention was given to familiarizing children with the language of arithmetic. Monroe (1918) discovered that of 108 types of problems met in his textbook survey, only 26 were common to all the books studied.¹⁶ The last five years have witnessed publication of a number of textbooks in arithmetic whose trend of planning evidently is to apply the definitely determined psychology of reading to arithmetic on the assumption that arithmetic would profit *per se*.

A minor study which has exerted influence upon teaching is that of Sara Chase¹⁷ (1917) conducted to determine sources of waste and obstruction in arithmetic. A useful outcome of the work is a list of words common to arithmetic with the percentages of the experimental group not understanding their meaning. A type of semi-scientific investigation much in use in corrective studies is the measurement and evaluation of the effect of definitely planned and explicitly described diagnostic and remedial programs. Of this character is the work of Estaline Wilson¹⁸ (1922) whose especial contribution Dr. Uhl expresses as "a specific example of providing interpretations and connections"¹⁹ through training in the specific reading ability required with arithmetic materials.

An investigation with probably better scientific control than is customary in the remedial study is that of Lutes²⁰

¹⁴ Ibid., p. 48.

¹⁵ Ibid., p. 8.

¹⁶ Monroe, Walter S., *The Derivation of Reasoning Tests in Arithmetic*, School and Society, VIII (1918).

¹⁷ Chase, Sara, *Waste in Arithmetic*, Teachers College Record, XVIII, 360.

¹⁸ Wilson, Estaline, *Improving the Ability to Read Arithmetic Problems*, Elementary School Journal, XXII (1922), 380.

¹⁹ Uhl, Willis H., *The Materials of Reading*, 1924, p. 253.

²⁰ Lutes, O. S., *An Evaluation of Drill Techniques for Improving*

(1926) who sought to make some analysis of the psychology of problem solving through comparison of drill techniques. Among his conclusions he states that practically nothing is known regarding the actual difficulty of problems found in arithmetic texts. The type arithmetic investigation most in evidence in recent years is, however, the study elaborately planned for the purpose of making critical analysis of pupils' errors or of causes of failure in problem solution, the findings being in most instances presented in general, often vague classification of types of error or of widely inclusive causes of failure. As would be expected, one notes a general similarity in the lists accruing from these error studies. An extensive study of reasoning errors made by Osburn ²¹ (1922) classifies the disabilities presumably underlying children's errors in problem work, the decision as to cause being, in some instances, in the absence of other clues, the inference of the investigator. Osburn's later study ²² (1925) lists nine causes of failure to understand the arithmetic problem. Studies of this type are probably best described as interpretative rather than quantitative. In 1925 C. S. Rice ²³ made an analysis of pupil difficulties with word problems in arithmetic. He does not discuss causes of failure because, as he explains, personal conferences with fifty individual pupils proved that they knew very little about why they solved the problems missed in the manner they had chosen. Mr. Rice locates no vocabulary weaknesses, his explanation being that Buckingham seems to have avoided technical terms in his tests. One recalls Monroe's statement that the principle applicable to the problem together with the meaning of the technical terms supplies the data for reflective thinking.²⁴ Rice comments

Ability to Solve Arithmetic Problems, University of Iowa Monographs in Education, Series I, Vol. 6, 1926.

²¹ Osburn, W. J., Diagnostic and Remedial Treatment for Errors in Arithmetical Reasoning, Wisconsin State Department of Public Instruction, 1922.

²² Osburn, W. J., Reading Difficulties in Arithmetic, Department of Education, Madison, Wisconsin, 1925.

²³ Rice, C. S., An Analysis of Pupil Difficulties with Word Problems in Arithmetic, M.A. Thesis, University of Chicago, 1925, unpublished.

²⁴ Monroe, Walter S., The Derivation of Reasoning Tests in Arithmetic, School and Society, VIII (1918), 297.

further that the trouble with the problems (Buckingham's Scale) was that situations were not real for children, that they were "only problems." This conclusion is a definite instance of the adopting of a mere assumption into one's dynamic thinking in the matter of the psychology of the verbal problem.

In a series of case studies ²⁵ made through careful analysis of children's test papers, Morton (1925) offers a typical classification of errors in problem solution. That arithmetic teaching is not rewarded by any considerable measure of success is implied by the fact that in the sixth grade 57.2 per cent of the errors fell under the classification "Procedure wholly wrong or entirely inadequate," a further cause being "Lack of knowledge of facts." The tendency to report these investigations in vague general terminology is exemplified by Stevenson's enumeration of five causes of failure ²⁶ located in a study of problem solving made with the cooperation of 32 elementary teachers (1925):

1. Physical defects
2. Lack of mentality
3. Lack of skill in fundamentals
4. Inability to read in general or to read arithmetic problems
5. Lack of general and technical vocabulary

Of different nature are those studies which seek to designate the mental factors or abilities involved in problem solving in arithmetic and to examine their operation. The specific problem ²⁷ set up for the two year study of Washburne and Osborne (1926) was the discovery of factors causing trouble in problem solving. The report of this study is introduced with the thesis that, "training children to solve arithmetic problems is one of the hardest and most discouraging tasks of the teacher. The pupils seem to have a way of doing the wrong thing, of simply juggling the numbers, that is most

²⁵ Morton, R. L., Papers in Educational Research Bulletin, Ohio State University, April 15 and 29, May 13, 1925.

²⁶ Stevenson, P. R., Difficulties in Problem Solving, Journal of Educational Research, February 1925.

²⁷ Washburne, C. W. and Osborne, Raymond, Solving Arithmetic Problems, Elementary School Journal, November and December 1926.

exasperating." After a period of serious experimentation answers to four questions set up for the study are given as follows:

1. There does not appear to be any difficulty in reading simple one step problems understandingly.

2. No relation has been found to exist between ability to make formal analysis of problems and ability to solve problems.

3. Unfamiliarity with materials and situations in problems is a small but significant factor in causing difficulty with problem solving. While the unfamiliarity of the situation enters in as a difficulty in problem solving, it is not so large an element as might be supposed.

Upon examination of the materials of this investigation, one finds that familiarity with the situation is not to be identified with interest in the situation.

To supplement this study, Dr. Washburne and Mrs. Moffett²⁸ (1928) devised a problem test to further investigate the matter of the unfamiliarity of the situation as an element of difficulty in solving problems. With eight pairs of problems (familiar and unfamiliar situations paired) it was found that in four pairs children made higher percentages with the familiar situations and that in the other four there was no significant difference. The text of the discussion would suggest that the discrimination between familiar and unfamiliar situations was made by designating as unfamiliar the situation which "seems *a priori* less familiar." Although the authors make specific recommendations, on the basis of these outcomes, for making the arithmetic situation "familiar and childlike," and such that, were the child "in the situation described in the problem, he would feel a real need for finding the solution," there seems to be little substantial justification in their data for the recommendations.

A generally scientific investigation, although only the

²⁸ Washburne and Moffett, Unfamiliar Situations as a Difficulty in Solving Arithmetic Problems, Journal of Educational Research, October 1928.

conclusions were published, is the quantitative study of eleven variables made by Morton ²⁹ (1925) "to account, in a degree, for the success of a group of pupils in solving verbal arithmetic problems." The data were subjected to elaborate statistical treatment through partial and multiple correlations. Certain of the zero order coefficients thus obtained have bearing on this study:

CORRELATIONS WITH ABILITY IN PROBLEM SOLVING (MORTON)

<i>Factor or Ability</i>	<i>Correlation with Ability in Problem Solving</i>
Verbal Intelligence.....	.78
Non-Verbal Intelligence.....	.52
Skill in Fundamental Operations.....	.70
Ability in Problem Analysis.....	.79
Reading Comprehension.....	.61
Reading Rate.....	.23
Marks in School Subjects.....	.33
Marks in Deportment.....	— .05
Attendance	.11
Age	.34

The coefficients yielded by these data are at variance with certain other investigations at some points. Washburne and Osborne found no relation to exist between ability to make formal analysis and the ability to solve problems. In a study of mathematical ability as related to general intelligence, Buckingham ³⁰ reports that there is low correlation between scores in tests of general intelligence and scores in arithmetic tests. It might be well to note also that reading scores in Morton's investigation were obtained on the Stone Narrative Reading Tests. Since Morton's thinking is of the careful scientific type, his comment (p. 135 thesis) is particularly significant:

Many writers have insisted that the problems supplied to pupils must be real and such as probably occur in the experiences of the pupil.

²⁹ Morton, R. L., Factors Affecting the Ability of Pupils to Solve Verbal Problems in Arithmetic, Ph.D. Dissertation, Ohio State University, 1925, unpublished. Manuscript loaned by the author. (For an account of the study see Dr. Morton's Teaching Arithmetic in the Intermediate Grades, Silver Burdett, 1927, p. 296.)

³⁰ Buckingham, B. R., Mathematical Ability as Related to General Intelligence, School Science and Mathematics, XXI (1925).

They have vigorously criticized the kinds of problems which the textbooks often contain. These criticisms, again, sound reasonable and one is inclined to believe that if they are heeded pupils will get along better in their efforts to solve problems, but, here too, experimental evidence has not been provided.

Myers⁸¹ (1925) reports an experimental study of "how problems designed to stimulate vivid imagination compare in difficulty with very similar problems of the dry, concise traditional sort." Twelve problems (6 pairs) served as material with the result that the imaginative problem was solved by a larger number of children in five cases out of six. In noting this study, Dr. Buswell⁸² comments that there are more issues involved in the general problem than are ordinarily realized and that objective evidence relating to these various issues is scarce at the present time.

Clapp and Hyde⁸³ (1927) made an extensive investigation to determine the extent to which the kinds of objects named in problems, or the kinds of objective settings described, determine their difficulty, it being the hypothesis of the experimenters that the child's visual imagery is a large factor in the interpretation of concrete problems. For each of eight elements of difficulty, two sets of problems were devised one as nearly like the other as possible except that "the element of difficulty was present in less measure." The complete experimental material was not administered to the entire group, a procedure having been devised to use two groups equated for arithmetic ability. Each division worked a share of the problems, the two groups being considered as a single group in the interpretation of the data. This study has been withheld from publication for two years. Now that it becomes available, however, certain of its findings are selected for their value in our own work:

1. *Objective Setting*—Objects easy to visualize (books, sheep, cars, overcoats) *vs.* Objects hard to visualize (tons of hay, quarts of cream, pounds of meat, miles, hours, weeks).

⁸¹ Myers, Garry C., *The Prevention and Correction of Errors in Arithmetic*, Plymouth Press, 1925, p. 65.

⁸² Buswell, G. T., *Summary of Arithmetic Investigations II*, *Elementary School Journal*, June 1928, p. 733.

⁸³ Clapp and Hyde, *Elements of Difficulty in the Interpretation of Concrete Problems in Arithmetic*, University of Wisconsin, Bureau of Educational Research Bulletin, No. 9, 1927, p. 13.

The objective setting proves to be an element of difficulty in that higher percentages of success were obtained with problems of the first type.

2. *Unfamiliar Objects*—"Not to be confused with the general diction and vocabulary of the problem." Quarts, yards *vs.* liters, meters; dollars and pounds *vs.* pesos and kilowatt-hours.

The percentage of error with unfamiliar objects is greater, the difference, however, being only 5 per cent.

3. *Non-Essential Elements*—In each instance this element introduces also a superfluous number which in our own thinking accounts, no doubt, for the material increase in difficulty accompanying inclusion of such elements found by these experimenters.

4. *Experience*—"A problem was thought to be within the experience of pupils when it represented a specific activity in which they might have actually participated, or an activity which they might have observed frequently." Contrary conditions are thought of as being "without pupils' experience."

A total of 6086 pupils participated in this phase of the study with the result that no consistent difference was found in the two types of problems.

5. *Symbols*—(A) seventy-five *vs.* (B) 75; (A) 400 X at \$5 per X *vs.* (B) 400 pairs of shoes at \$5 a pair.

In grades 4 and 5 the total percentage of error in A exceeds that in B by less than 4 per cent, there being practically no difference in the difficulty of the contrasted types in grades 6, 7, and 8.

Monroe³⁴ (1929) reports an extensive study conducted in seventh grade with the inclusion of a few sixth and eighth grade classes to determine how pupils solve problems in arithmetic, the thought of the author being that the nature of pupils' responses to changes in the statement of a problem would furnish an indication of how they proceed in solving it. It is important in considering the outcomes to have in mind the author's own defining of the factors varied:

A statement of a problem was considered abstract if there was no reference to the activity in which it occurred. If this activity was indicated in the statement it was called concrete. The terminology of a problem was considered technical when the terms used were those that are commonly employed by specialists in a particular field. The terminology was considered to be simple when it consisted of words and phrases in general use. A third modification in the statement of a problem was secured by introducing irrelevant data.

As with Clapp, Monroe used similar problems but in his case four groups of children shared the test material since, as he says, it would not be satisfactory to have the same pupils

³⁴ Monroe, Walter S., *How Pupils Solve Problems in Arithmetic*, University of Illinois Bulletin, No. 44, 1929, p. 7.

respond to two or more statements of the same problem. That the data from the four groups are treated as though they were obtained from a single group lays the procedure open to the same criticism, perhaps, that Clapp mentions as having been made concerning his having used two groups. It is probably not possible in an experiment of this character to control all the variables as one might wish. In the present study it has seemed the more reliable procedure to vary the problems and to have all the children solve all the problems.

Monroe's conclusions are as follows:

1. In general the data support the thesis that the use of technical terminology increases the difficulty of arithmetical problems, but the exceptions indicate that such terminology does not necessarily make a problem more difficult.

2. The presence of irrelevant data makes the problem more difficult, although several of the differences are small.

(It is essential to note that upon examination of these problems one finds the irrelevant data invariably including a superfluous number.)

3. The introduction of phrases descriptive of a concrete setting for a problem makes little or no difference in the responses of pupils. They respond to abstract problems about as satisfactorily as they do to those having a concrete setting.

Dr. Monroe's interpretation of his findings is, in part, that children do little or no reflective thinking in solving arithmetic problems; that many of them appear to perform almost random calculations upon the numbers given, and that when they do solve a problem correctly, the response seems to be determined largely by habit.

Wheat⁸⁵ (1929) conducted a study to determine which type of arithmetic problem, the conventional or the imaginative, possesses the greater value as an exercise to aid in generalizing knowledge of the fundamental operations. Wheat's own definitions of the terms *conventional* and *imaginative* are important:

⁸⁵ Wheat, Harry G., *The Relative Merits of Conventional and Imaginative Types of Problems in Arithmetic*, Contributions to Education, No. 359, Columbia University.

The term 'conventional' is used to describe the type of problem in arithmetic which is stated in the shortest, simplest, and most direct language possible. The conventional problem contains no mention of, nor allusion to, any element of the total situation under consideration—however interesting or appealing such element may be—other than those which may be described as the mathematical elements, or which require mathematical terms for their portrayal, or demand mathematical thinking for their complete understanding. . . . The term 'imaginative' is used to describe the type of problem in arithmetic which contains elements of the total situation which are interesting and have an imaginative appeal, but whose presence is not germane to the problem under consideration nor necessary for an understanding of the mathematical elements of the situation presented.⁸⁶

The differentiation in Wheat's study is on a basis of details and not on that of children's interests. The problem situations are described as being those, "which are familiar to boys and girls in the fifth grade of the elementary school. They are the kind of situations with which such pupils have become familiar in part through actual experience and in part through contact in the daily problem practice in their work in arithmetic." Wheat undertook this investigation to secure further experimental evidence concerning Dr. Myers's claim to having established a case for the longer problem:

No question is raised in these reports (on Myers's work) as to whether the experimentation upon the relative merits of the two types of problems has been extensive enough or careful enough to warrant comment and recommendation conclusively favorable to either type.⁸⁷

Using test material which incorporates Myers's problems, Wheat controverts the previous findings. His data indicate that pupils of the intermediate grades are neither hindered nor helped in their problem practice exercises by problems of the imaginative type when no limits are imposed upon the amounts of time of the practice periods. Since these two investigators disagree, the matter, it would seem, is still debatable. Moreover, Wheat's material indicates, as does Myers's, the assumption, unsupported by experimental evidence, that the declarative form of statement is to be preferred to the interrogative form. At least, this inference might reasonably be made from the fact that of about sixty problems used by Wheat only three are in interrogative form.

⁸⁶ Ibid., p. 1.

⁸⁷ Ibid., p. 4.

Bowman³⁸ (1929) made a study in junior high school to determine whether pupils are more successful in performing arithmetic problems for which they express a preference. The reported preference was found to be correlated with performance to the degree of .56.

It would seem, after this brief survey, that scientific evidence concerning the arithmetic problem is more limited and less sanguine than might have been expected. While several recent scientific investigations have found that variation of certain factors resulted in little material difference in children's achievement, the present trend of inquiry is significant. The test materials used in these studies have been devised entirely subjectively, but certain of the experimenters are of such experience that their opinions in these matters are to be reckoned with. The study submitted in the following pages has used the objective criteria available. Its problem embraces a narrow range of investigation not previously attempted but the outcomes will be found to be not at variance with related studies.

³⁸ Bowman, H. B., *The Relationship of Reported Preference to Performance in Problem Solving*, University of Missouri Bulletin, No. 36, 1929, p. 49.

CHAPTER III

PLAN OF THIS STUDY

THE EXPERIMENTAL TESTS

Each of the four factors under investigation is studied in two contrasting forms:

I. Sentence Form of the Problem

a. Interrogative-Complex

The entire problem stated within the confines of a single complex-interrogative sentence, the condition or supposition being set up by an adverbial clause, which is in most instances an inverted clause in "if."—Atmosphere of suspension.

b. Declarative

The facts of the problem stated in a declarative, or series of declarative sentences, the requirement being given in a further interrogative or imperative sentence.

II. Vocabulary of the Problem

a. Unfamiliar

b. Familiar

III. Style of the Problem

a. Brief, compressed, without details

b. Relevant details (no superfluous numbers)

IV. Problem Situation

a. Uninteresting

b. Interesting

Every problem must of necessity embody each of these four factors in one of the two contrasting forms. It was the hypothesis of this study that the *a* form is the difficult form and that the combination of the four factors in their *a* forms would constitute the most difficult problem form. The problem whose characteristics are expressed by suspension (*a*),

unfamiliarity of vocabulary (*a*), compression (*a*), and uninteresting situation (*a*) is given the code *a a a a* and is designated Form 1.

To provide for possible permutations, it was necessary to vary the factors of this combination one, two, three and finally four at a time. The plan for the evolution of the 16 forms is as follows, the legend noting variations from Form 1 as indicated by occurrence of *b* in the code:

UNINTERESTING		
Form	Code	Legend
1	a a a a	Interrogative-Complex
2	b a a a	Declarative (<i>b</i>)
3	a b a a	Interrogative-Complex—Familiar Vocabulary (<i>b</i>)
4	b b a a	Declarative—Familiar Vocabulary
5	a a b a	Interrogative-Complex—Details (<i>b</i>)
6	b a b a	Declarative—Details
7	a b b a	Interrogative-Complex—Familiar Vocabulary—Details (<i>b b</i>)
8	b b b a	Declarative—Familiar Vocabulary—Details
INTERESTING		
9	a a a b	Interrogative-Complex—Interesting (<i>b</i>)
10	b a a b	Declarative—Interesting (<i>b</i>)
11	a b a b	Interrogative-Complex—Familiar Vocabulary—Interesting
12	b b a b	Declarative—Familiar Vocabulary—Interesting
13	a a b b	Interrogative-Complex—Details—Interesting
14	b a b b	Declarative—Details—Interesting
15	a b b b	Interrogative-Complex—Familiar Vocabulary—Details—Interesting
16	b b b b	Declarative—Familiar Vocabulary—Details—Interesting

This sequence of 16 forms provides for two contrasting sets of 8 forms each for each of the four experimental factors:

I. Sentence Form

Interrogative-Complex	Forms 1, 3, 5, 7; 9, 11, 13, 15
Declarative	Forms 2, 4, 6, 8; 10, 12, 14, 16

II. Vocabulary

Unfamiliar Vocabulary	Forms 1, 2, 5, 6; 9, 10, 13, 14
Familiar Vocabulary	Forms 3, 4, 7, 8; 11, 12, 15, 16

III. *Style*

Brief, Compressed
With Details

Forms 1, 2, 3, 4; 9, 10, 11, 12
Forms 5, 6, 7, 8; 13, 14, 15, 16

IV. *Content*

Uninteresting
Interesting

Forms 1, 2, 3, 4, 5, 6, 7, 8
Forms 9, 10, 11, 12, 13, 14, 15, 16

NUMBER CONTENT

Since this study was conducted in the 6 B grade, the arithmetical content of the test material employs subject-matter of the 5 A grade, which in Baltimore comprises common fractions and practice work with integers. The operations required included long division; the various cases of division with common fractions, omitting division of a proper fraction by a similar fraction; partitive multiplication; and the multiple operation with common fractions, involving, as was the case in division, proper fractions, integers, and mixed numbers. The problems were scored for principle. The number complexity of the uninteresting material is about equivalent to that of the interesting material, such difference as exists probably rendering the latter slightly the less difficult. With two exceptions, the denominators used in fractions were not greater than 8, and in almost 80 per cent of the instances, the denominator was either 2, 4, or 8. The problems were invariably limited to a single operation, one step problems, as the type is commonly known.

THE TEST FORMS

One hundred and twenty problems, 60 interrogative-complex and 60 declarative, were devised on the various pattern-forms. In the refinement of the data, for each of the 16 forms there were selected the 6 problems which best conformed to the criteria for the form in question. This selected material, comprising 96 problems (48 division and 48 multiplication), provides 6 trials for each of the 16 pattern-forms.

It embodies, primarily, 48 problems for each of the contrasting conditions of the four factors under study:

a. Interrogative-Complex	48
b. Declarative	48

a. Unfamiliar Vocabulary.....	48
b. Familiar Vocabulary.....	48
a. Without Details.....	48
b. With Details.....	48
a. Uninteresting	48
b. Interesting	48

In addition to the 120 problems described, the test material included 8 other problems, most of them expressed in a single interrogative sentence. Although excluded from computations for pattern-forms and factor effect, this smaller group serves to press the inquiry further at certain points.

The 128 problems were divided into 8 tests of 16 problems each, effort being made at the time of the division to so allocate pattern-forms that each would receive somewhat fair distribution of position in the consecutive tests.

SCORING

Problems were scored for principle right, the small number of cases in which numerically correct answers were given without the work being included in this category. The test papers were further meticulously analyzed with reference to a scoring scheme approved by Professor Bamberger. Each test was scored twice, there being 2515 test papers. The outcomes of the study are presented through comparisons of *Percentage Right*. The eight tests are given in the appendix with a notation for percentage right for each problem.

THE EXPERIMENTAL GROUP

The experimental group comprised 6 B classes, the first semester of sixth grade, in eight elementary public schools in Baltimore. The schools selected insure a representative sampling of economic conditions prevailing in our school population:

1. Fine residential section—No. 233 Roland Park
2. Good residential section—No. 63 Walbrook
3. Average residential section—No. 86 Mulberry and Payson Streets
4. Average residential section—No. 99 North Avenue and Washington Streets
5. Poor residential section—No. 237A Highlandtown
6. Poor residential and industrial section—No. 98 Ashton and Pulaski Streets

7. Industrial tenement district—No. 3 Eastern and Montford Avenues

8. Industrial tenement and business district—No. 93 Central Avenue and Lexington Street

Of the pupils enrolled in these eight classes, 237 (108 girls and 129 boys) were present each of the eight consecutive days of the testing. This constant group of 237 children constitutes the experimental group, all of the children having taken all of the tests.

That the group might be typical in academic ability, classes were chosen with reference to distribution of intelligence as evidenced by Class Analysis Charts on file in the Bureau of Research, the Illinois Intelligence Examination having been given on these pupils' entrance to 5 B. The average I.Q. for the experimental group is 101.42 with S.D. 18.14. That the group is representative is indicated by the fact that for 16,138 Baltimore children, grades III to VIII, measured with the same test the average I.Q. is 94.3 with the almost identical S.D. of 18.1.

In sectioning for X, Y and Z groups we have, at Dr. Brooks's suggestion, used the following levels designated by Dr. Terman, rather than those then in use in Baltimore:

Z — I.Q. less than 90 — 62 cases

Y — I.Q. 90-110 — 95 cases

X — I.Q. 110 and over — 80 cases

In the few instances in which this thesis selects a single subgroup for critical study, the group chosen has been the X group (I.Q. 110 or over).

A third factor influencing the selection of classes for the study was the class achievement as indicated by the class median in the Baltimore Course of Study Test administered in the city-wide testing program three months previous to this experimental work. The median of medians at that time (February 1928) for 125 classes, 6 B grade, was 16.227 with Q. 3.238. For the experimental group the average is 16.18 with S.D. 8.03. When this course of study test was repeated in February 1929, the S.D. for 1178 pupils of the 6 B grade was 7.66. In arithmetic ability, then, that is in those specific abilities measured by this course of study test, the selected

group is a fair and typical sampling of 6 B children in Baltimore.

The mean age of the experimental group is 11 years—10.8 months with S.D. 11.9 months. Since the normal pupil has his twelfth birthday in the sixth grade, this mean age is typical for children who had been three months in 6 B.

As measured by the means available, the selected group is, then, in these factors significant to the investigation, a typical sample, a cross section, as it were, of the total 6 B enrollment in Baltimore's public schools:

1. Economic Distribution
2. Academic Ability
3. Arithmetic Ability
4. Chronological Age

GIVING OF THE TESTS

The tests were printed on the linotype in eight forms, space being arranged at the right of the problem for the pupil's work. They were given first period each morning for eight consecutive days, May 4-15, 1928, inclusive. In two instances the tests were administered by principals experienced in testing; in the six other schools the testing was conducted by the Local School Examiner connected with the Bureau of Research.

RELIABILITY OF THE TESTS

In determining the reliability of the tests, it was found that to section the tests into the first four forms and the second four forms would divide the entire material into comparable halves. The Spearman-Brown¹ formula for the coefficient of reliability was then applied with the scores made in the two halves by 125 boys and girls constituting the upper 53 per cent of the distribution for intelligence. That the coefficient of reliability is $.929 \pm .008$ is evidence that the tests are a reliable measure of the abilities in question.²

¹ Kelley, *Statistical Method*, p. 206.

² Garrett, *Statistics in Psychology and Education*, p. 270.

CHAPTER IV

FINDINGS OF THIS STUDY

SEX DIFFERENCES IN ACHIEVEMENT

In total achievement the boys' average exceeds that of the girls by .8 per cent, or, to use the common measure, 50.4 per cent of the boys reach or surpass the girls' median of 63.5 problems, the possible score being 120. This result is in harmony with the study of Fox and Thorndike¹ (1903) which found sex differences in arithmetic so small as to be negligible. In summarizing six studies in sex differences in arithmetic ability, Professors Judd and Buswell² conclude that these differences are small, those which were found being in favor of boys. In Dr. Terman's study³ of 565 gifted children, the mean I.Q. for the boys and girls being in each case 151.6, it was found that the mean arithmetic quotient of the boys exceeded that of the girls by only 2.8 (138.5—135.7). The expectation that boys will do better work in arithmetic than girls would seem, therefore, to be based upon tradition rather than upon fact.

INTELLIGENCE SCORES AND EXPERIMENTAL ARITHMETIC SCORES

Between success in problem solution and I.Q. (intelligence quotient) these data yielded a Pearson r of $.386 \pm .037$ with k .922. This measure is stabilized by Dr. Buckingham's having found in summarizing studies of relation between general intelligence and mathematical ability⁴ (1921) that the coefficient seldom reaches .40 and that the correlation between arithmetic scores and scores in general intelligence is also

¹ Fox and Thorndike, *The Relationships between the Different Abilities Involved in the Study of Arithmetic. Sex Differences in Arithmetic Ability*, Columbia University Contributions to Philosophy, Psychology, and Education, XI, February 1903.

² Buswell and Judd, *Summary of Educational Investigations Relating to Arithmetic*, University of Chicago, 1925, p. 134.

³ Terman, L. W., *Genetic Studies of Genius*, Vol. I, p. 291.

⁴ Buckingham, B. R., *Mathematical Ability as Related to General Intelligence, School Science and Mathematics*, XXI (1921), 205.

low. Analysis of the data of the present study would indicate, however, that the low correlation might possibly be interpreted as a measure of lack of success in developing children's reasoning ability in arithmetic, rather than as an indication of absence of more definite relation between potential arithmetic ability and general intelligence.

CHRONOLOGICAL AGE AND THE EXPERIMENTAL TESTS

Between chronological age and ability in problem solution, the research material shows a negative correlation, the r being $-.199 \pm .042$ with $k .98$. It will be recalled that the average age in this group is 11 years and 11 months with S.D. 11.9 months. It is a common fallacy among teachers to hesitate to provide for acceleration among the younger and brighter children because of the fear that they lack the experience necessary for accomplishment in arithmetic. With this group, age does not seem to be a factor predictive of success with arithmetic problems.

COMPUTATION AND THE RESEARCH TESTS

Between the scores in the experimental tests and the scores in the Baltimore Arithmetic Course of Study Test, given at the beginning of the semester, the Pearson r is $.598 \pm .028$ with $k .801$. Examination of the Course of Study Test would probably confirm its classification as a test in the fundamentals of arithmetic, there being but two verbal problems among 34 test items. The interval of time elapsing between these two tests is to be regretted, but class time could not be secured for further testing in May. Dr. Morton⁵ secured an r of .70 between arithmetic skills and problem solving. Dr. Thorndike⁶ is of the opinion, however, that were we to test a thousand fourteen-year-old children with enough tests to measure each individual exactly in both problem solving and computation, the correlation would probably not be over .60. The r of .598 yielded by the data of our study may be

⁵ Morton, R. L., *Teaching Arithmetic in the Intermediate Grades*, 1927, p. 297.

⁶ Thorndike, Edward L., *The Psychology of Arithmetic*, 1922, p. 299.

accepted, therefore, as a somewhat secure indication of the real relation existing between these two types of arithmetic ability with this group of children. The data for all correlation computations are given in the appendix of the manuscript copy of the investigation on file at The Johns Hopkins University.

THE EFFECT OF INTEREST

The effect of each of the four factors previously discussed is presented on the basis of data accruing from the best possible selection, in accordance with the various criteria, of six sets of material. The computations are for six trials for each of the sixteen pattern-forms with consequent equal amounts of contrasting material from the standpoint of each of the four factors: interest, sentence structure, details, and vocabulary. Percentages express the relation of rights to attempts, the original data being used in every instance to secure totals in preference to taking an average of percentages.

TABLE I
COMPARISON OF SUCCESS—INTERESTING AND UNINTERESTING MATERIAL

	Uninteresting			Interesting			Diff. %
	Attempts	Rights	Per Cent	Attempts	Rights	Per Cent	
Entire Group..	10843	5382	49.64	10961	5508	50.25	0.61
X — Bright Pupils.	3718	2097	56.40	3726	2161	58.00	1.60
Y — Average Pupils	4360	2148	49.27	4415	2171	49.17	— 0.10
Z — Dull Pupils...	2765	1137	41.12	2820	1176	41.70	0.58

To measure the reliability of the obtained difference between uninteresting and interesting material for the entire group, averages were calculated for the distributions of percentages of success for the 48 individual problems in each group, in order to apply the formula for the sigma of the difference.⁷ It will be noticed that this sigma is small.

$$\begin{aligned}\sigma(\text{diff.}) &= \sqrt{\sigma^2(\text{av. 1}) + \sigma^2(\text{av. 2})} \\ &= .048 \\ \text{diff.} &= .0032 \pm .048\end{aligned}$$

The differences found for the various factors by these distributions of individual percentages to compute the sigma

⁷ Garrett, H. E., *Statistics in Psychology and Education*, p. 129.

will, of necessity, not be identical with differences found by comparing percentages obtained from total rights over total attempts.

It was the aim to incorporate interest for both boys and girls in the interesting problem situations. As would be expected, however, certain of the problems make stronger appeal to boys, or to girls, as the case may be. Tables II and III offer percentages of success for boys and girls separately with two small groups of problems. There is less than one per cent of difference in each case, the boys exceeding the girls by but .8 per cent in problems more characteristic of boys and by .9 per cent in the small group of problems offering greater interest appeal to girls.

TABLE II
GROUP OF PROBLEMS PRESENTING STRONGER INTEREST APPEAL
TO BOYS THAN TO GIRLS

	Percentage of Success	
	Boys	Girls
1	67.2	64.4
2	22.1	14.4
3	46.8	44.7
4	67.7	66.0
5	72.1	67.2
6	13.1	9.6
7	61.9	62.9
8	52.0	52.0
9	16.1	16.5
10	67.7	74.5
11	94.4	87.0
12	66.7	72.3
13	59.7	64.4
Average	54.5	53.7
Diff.		0.8% On original data In favor of boys

TABLE III
GROUP OF PROBLEMS PRESENTING STRONGER INTEREST
APPEAL TO GIRLS

	Percentages	
	Boys	Girls
1	9.4	12.3
2	63.2	65.1
3	73.2	69.8
4	69.3	68.5
5	67.2	58.1
6	15.9	17.8
Average	49.5	48.6
Diff.		0.9% In favor of boys

PROBLEM WORK IN DIVISION

Analyses are given later to locate causes of the children's startlingly poor achievement with problems involving the principle of division. The purpose of Table IV is to contrast success in selection of principle with interesting and uninteresting problems in division.

TABLE IV
COMPARISON OF SUCCESS WITH UNINTERESTING AND INTERESTING
PROBLEMS INVOLVING THE PRINCIPLE OF DIVISION

	Uninteresting			Interesting			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5394	2093	38.80	5441	2086	38.34	—0.46
Bright	1842	843	45.77	1841	811	44.05	—1.72
Average	2171	825	38.00	2190	835	38.13	0.13
Dull	1381	425	30.78	1410	440	31.21	0.43

The differences are again negligible except in the case of the bright group where the achievement is about 2 per cent better with uninteresting material.

PROBLEM WORK IN MULTIPLICATION

Table V submits corresponding data for the problems involving the principle of multiplication.

TABLE V
COMPARISON OF SUCCESS WITH INTERESTING AND UNINTERESTING
MATERIAL IN MULTIPLICATION

	Uninteresting			Interesting			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5449	3289	60.35	5520	3422	61.99	1.64
Bright	1876	1254	66.84	1885	1350	71.62	4.78
Average	2189	1323	60.44	2225	1336	60.04	—0.40
Dull	1384	712	51.45	1410	736	52.20	0.75

Comparison of the percentages of success for the several groups leads to the conclusion that the application of the principle of multiplication in problem solution is much more readily discerned than is the division principle. In this process, moreover, the entire group achieves 2 per cent greater success with interesting material, the difference being due to the contribution of the bright group in which the gain

amounts to 5 per cent. The difference is again, as in division, less than one per cent for the dull children and almost imperceptible with the average group. Table VI assembles the outcomes in terms of gains for interesting material.

TABLE VI

EFFECT OF INTERESTING MATERIAL UPON PROBLEM SOLUTION
EXPRESSED IN GAINS OVER UNINTERESTING MATERIAL

	Division	Multiplication	Total Test Material
	%	%	%
Entire Group	-0.46	1.64	0.61
Bright	-1.72	4.78	1.60
Average	0.13	-0.40	-0.10
Dull	0.43	0.75	0.58

* The conclusion is, then, that interesting material based on children's very carefully determined interests fails to induce keener or more successful arithmetical thinking, at least to an appreciable or actually useful extent. The tendency is in favor of interesting material, the increase in the case of bright pupils reaching 1.6 per cent. In the more difficult principle of division, however, the superior group achieves 1.7 per cent less success with interesting problems, its achievement with the more readily discerned multiplication principle being 4.8 per cent higher with the interesting material. The average group makes no perceptibly improved reaction to the factor of interest inherent in the material. In fact, only in the case of division does the variation amount to one per cent. Although a variety of causes may actuate omission of test items, the finding is, perhaps, somewhat further stabilized by the fact that for the entire group there is little difference between the number of problems omitted in the two types of material, an average per pupil of 2.2 uninteresting and 1.8 interesting problems.

MEANING OF THE FINDING CONCERNING INTEREST

This finding would seem significant in arithmetic teaching. The assumption prevalent with textbook makers is that to base problems upon children's home and school activities, upon their plays, games, and social activities, will insure

better learning. The school has responded to the urge to employ interesting textbook material. Doubtless, to regard children's interests in our teaching and selection of subject matter is good psychology. The implication of this finding is that, "other things being equal," interesting material does not of itself produce better results in arithmetic.

The school has come, probably, to rely too much upon the potency of the factor of interest and to overlook the fact that arithmetic is essentially the most difficult of school subjects. It is a mode of thinking into which children seldom venture of themselves. Among educators there are two groups of thinkers in the field of arithmetic. One would limit the content of arithmetic to that demanded by social usage. The other regards it as a subject furnishing opportunity for cultivation of useful mental habits. Both groups would probably agree that children should be carefully trained to locate intelligently the need or requirement of the arithmetic problem and to use with intelligence the facts which furnish the essential data for the solution. That children are interested in the problem situation from the imaginative standpoint does not insure arithmetical interest. Few adults are interested even to estimate quantitative relations existing between the numbers, dimensions, and so on, which occasionally occur in reading matter. To be imaginatively interested is not necessarily to be arithmetically interested.

The importance of this finding is, then, that the school must not rely upon the probable fallacy that interesting material of itself will improve children's work. There is little evidence in these data that children prefer their arithmetic to concern itself with their plays, games, and social activities. They did about as well, in fact, with traditional materials of adult industrial and economic concern as they did with carefully selected children's material. The business of arithmetic is to train children in critical, exact, precise thinking with arithmetical data. That these data were incorporated into interesting material did not insure improved thinking on these children's part. We must teach children to think arithmetic if arithmetical thinking is to result.

This outcome in regard to the effect of interest is in harmony with Monroe's conclusion⁸ (1929) that children "respond to abstract problems about as well as they do to those having a concrete setting." It agrees also with the finding of Hydle and Clapp⁹ (1927). It will be recalled that after experimenting with problems within children's experience but with objective settings difficult to visualize and problems without their experience but with objective settings easy to visualize, their conclusion is that "the results indicate no consistent difference in the difficulty of the two types of material."¹⁰ Both of these studies base their differentiation of contrasting materials upon the judgment of the experimenters, men in each case, however, of research experience in the field of arithmetic.

The present study has assembled available criteria of children's interests to guide in the formulation of interesting arithmetic problems. Small differences in achievement have resulted with the two types of material with little evidence to encourage the popular presumption that, other things being equal, children will attain greater success with interesting arithmetic material. Small differences are, however, significant in this case in that they fail to support the uncritical hypothesis gaining mastery in arithmetic teaching that school children will work better with interesting material. An arithmetic reviewer commends certain authors for having "written many a problem which the child will read for mere enjoyment,—a rare quality in arithmetic."¹¹ The enjoyment does not, in this study, appear to make the reasoning easier. It may, however, make the problems more desirable. The point is here that, according to these findings, interest does not initiate substantially improved arithmetical thinking.

⁸ Monroe, Walter S., *How Pupils Solve Problems in Arithmetic*, University of Illinois Bulletin, No. 44, 1929, p. 15.

⁹ Hydle and Clapp, *Elements of Difficulty in the Interpretation of Concrete Problems in Arithmetic*, Bureau of Educational Research Bulletin No. 9, University of Wisconsin, 1927, p. 36.

¹⁰ *Ibid.*, p. 87.

¹¹ Myers, Garry C., *Textbooks in Arithmetic (Review)*, Elementary School Journal, October 1927, p. 150.

THE EFFECT OF SENTENCE FORM

The following tables submit outcomes for two contrasting forms of problem statement. In the interrogative-complex form the entire problem is set forth within the confines of a single interrogative sentence. In the declarative form the facts of the situation are given by means of the declarative sentence, the requirement being proposed in a further interrogative or, in some cases, an imperative sentence. The problem was to discover which of these forms facilitates children's reasoning. The problems cited will serve as samples of the contrasting forms:

If a merchant tailor allows $3\frac{3}{8}$ yd. of double width material for an overcoat, what amount of this width material must he purchase to fill an order for 140 overcoats?

A fruit grower raising walnuts for a foreign market finds the average yield of an acre in his grove to be $96\frac{1}{2}$ bu. Find the total yield of 27 acres.

Clapp Drill Book in Arithmetic, Fifth Year, Silver Burdett, 1926 (Adapted from problem 2, p. 94).

TABLE VII

COMPARISON OF SUCCESS—INTERROGATIVE-COMPLEX
AND DECLARATIVE SENTENCE FORMS

	Interrogative-Complex			Declarative			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	10889	5472	50.26	10915	5418	49.64	—0.62
Bright Pupils	3706	2155	58.15	3738	2103	56.26	—1.89
Average Pupils	4389	2166	49.35	4386	2153	49.09	—0.26
Dull Pupils	2794	1151	41.20	2791	1162	41.63	0.43

TABLE VIII

COMPARISON OF FORM OF STATEMENT—DIVISION

	Interrogative-Complex			Declarative			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5400	2116	39.19	5435	2063	37.96	—1.23
Bright	1825	850	46.58	1858	804	43.27	—3.31
Average	2183	843	38.62	2178	817	37.51	—1.11
Dull	1392	423	30.39	1399	442	31.59	1.20

TABLE IX
COMPARISON OF FORM OF STATEMENT—MULTIPLICATION

	Interrogative-Complex			Declarative			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5489	3356	61.14	5480	3355	61.22	0.08
Bright	1881	1305	69.38	1880	1299	69.10	-0.28
Average	2206	1323	59.97	2208	1336	60.51	0.54
Dull	1402	728	51.93	1392	720	51.73	-0.20

There is no significant difference between success in the two forms of statement in multiplication. In division, in which the difficulty evidently deepens for children, there is a difference of one per cent in favor of the interrogative-complex form for average children, and *vice versa* for the dull. The bright children succeed better by 3 per cent with the interrogative form. For the entire group there is a difference of one per cent in favor of the interrogative-complex form in division. With the entire test material the bright children make a gain of about two per cent with the interrogative form and the entire group a gain approaching one per cent (0.6%).

Applying the formula for the sigma of the difference to the sigmas of the obtained averages of the distributions of percentages of success in the two sets of material, we find for the entire group that the reliability of the difference is as follows:

$$\sigma \text{ difference} = .048$$

$$\text{difference} = .0105 \pm .048 \text{ (In favor of the interrogative-complex statement)}$$

Because of the predominance in textbook material of the interrogative-complex form introduced by the inverted conditional clause in "If," this form was used in 90 per cent of the interrogative-complex material. It is interesting to note that the preference for the interrogative over the declarative form is slightly more evident with the "inverted if" problem.

	Interrogative-Complex	"Inverted If"	Difference
Entire Group	50.3%	51.7%	1.4%
Bright Group	58.2%	59.7%	1.5%

That the inverted interrogative sentence in "if" is not given a frequency rating or relative scale value in Dr. Thorndike's

inventory of English constructions¹² is explained by the omission of arithmetic texts from the material examined. It is the most common construction of the traditional arithmetic and is not yet routed by the declarative form. The changing attitude toward the form is expressed, however, in a direction to teachers in our own Baltimore course of study in arithmetic:¹³

The language of the problem must be simple and direct, avoiding the hypothetical 'if,' and obscure or technical terms.

It may be, however, that practice effect is operating in favor of the usual "inverted if" form and that training in the declarative form would yield results not found in this testing.

THE EFFECT OF DETAILS

It was found by Hydle and Clapp¹⁴ (1927) and Monroe¹⁵ (1929) that the inclusion of non-essential details which introduce also superfluous numbers increases the difficulty of arithmetic problems. Myers¹⁶ (1925) reports an experiment with a small amount of material in somewhat loose and extreme style in which he finds evidence in favor of the long problems making an imaginative appeal.

Our study compares two definite styles of expression, one the bare, compressed statement of the structure of those facts which furnish data essential for the solution of the problem; the other, the statement including relevant but unnecessary details. The problems are all one step problems and no superfluous numbers are given. In the uninteresting section the details are merely pertinent to the situation while in the interesting material they intensify the imaginative appeal.

It will be noticed from Tables X, XI, and XII that the gains for problems *with details* are consistently negative. In

¹² Thorndike, E. L., An Inventory of English Constructions with Measures of their Importance, Teachers College Record, February 1927.

¹³ Baltimore Course of Study in Arithmetic for Intermediate Grades, 1924 edition, p. 62.

¹⁴ Hydle and Clapp, Elements of Difficulty in the Interpretation of Concrete Problems in Arithmetic, 1927.

¹⁵ Monroe, Walter S., How Pupils Solve Problems in Arithmetic, 1929.

¹⁶ Myers, Garry C., The Prevention and Correction of Errors in Arithmetic, Plymouth Press, 1925.

division problems, all groups do better work with the brief statement of essential facts. The differences are small but experimentation recognizes the significance of small differences. As with the previous factors, the greater sensitivity is manifested in division. The bright group does less well by 2 per cent with details; the difference reaches about 3 per cent with the average and 1.3% with the dull group. The entire group shows a negative reaction of 2 per cent to the introduction of details in division problems.

TABLE X
COMPARISON OF SUCCESS WITH AND WITHOUT DETAILS
IN THE PROBLEMS

	Without Details			With Details			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	10931	5560	50.87	10873	5330	49.02	—1.85
Bright Pupils	3722	2147	57.68	3722	2111	56.72	—0.96
Average Pupils	4396	2231	50.75	4379	2088	47.68	—3.07
Dull Pupils	2813	1182	42.02	2772	1131	40.80	—1.22

TABLE XI
COMPARISON OF SUCCESS WITH AND WITHOUT DETAILS
DIVISION

	Without Details			With Details			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5421	2147	39.61	5414	2032	37.53	—2.08
Bright	1841	846	45.95	1842	808	43.87	—2.08
Average	2180	858	39.36	2181	802	36.77	—2.59
Dull	1400	443	31.64	1391	422	30.34	—1.30

TABLE XII
COMPARISON OF SUCCESS WITH AND WITHOUT DETAILS
MULTIPLICATION

	Without Details			With Details			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5510	3413	61.94	5459	3298	60.41	—1.53
Bright	1881	1301	69.17	1880	1303	69.31	0.14
Average	2216	1373	61.96	2198	1286	58.51	—3.45
Dull	1413	739	52.30	1381	709	51.34	—0.96

In multiplication the reaction against details reaches about 3.5 per cent for average children and about one per cent for the dull, there being no perceptible difference with the bright

pupils. The total reaction against details in this process is 1.5 per cent.

With the entire material the case is for the brief problem simply stated, its gain over the problem with details being one per cent for both bright and dull children and 3 per cent for average children. For the entire group the gain is 1.85 or about 2 per cent. In order to apply the formula for the sigma of the difference, the sigmas of the averages were computed. The two sets of material were redistributed for this purpose in terms of percentage right for individual problems within each section.

$$\begin{aligned}\sigma \text{ diff.} &= .049 \\ \text{diff.} &= .0177 \pm .049 \text{ (In favor of the problem} \\ &\quad \text{without details)}\end{aligned}$$

The hypothesis that the arithmetic problem should approach the story form is not supported by these data. These children did consistently better work with the problem presenting in concise form only the facts essential for the solution. It may be that a teaching experiment employing both types of material would yield more significant differences.

THE EFFECT OF VOCABULARY

The effect of unfamiliar vocabulary in contrast with familiar vocabulary in the arithmetic problem has been studied in this investigation with two separate measures, one the indication of word familiarity from the cumulative evidence of fifteen criteria (see Appendix II of the copy of this study previously referred to in The Johns Hopkins University library) and the other the scale position of words in the Thorndike *Word Book*. Tables are given for the outcomes resulting from the *seriatim* grading of the problems according to the writer's best use of the indication of word familiarity afforded by the cumulative evidence of fifteen vocabulary studies. These comparisons are for the entire group and for the bright group with the two halves of the material as determined by this vocabulary measure.

Table XIII shows positive differences of less than 2 per cent for each group in favor of familiar vocabulary. There is a positive gain of 4 per cent for the entire group and of

about 5 per cent for bright children in the case of multiplication. The fault is with the measuring rod. In most vocabulary studies we find investigators rating and re-rating the first few thousand common words so that we can be sure of the relative ease of about 3000 words. Beyond this level we are confronted by the absence of scales for determining word familiarity for children. There is, in fact, little reliable word measurement beyond primary material.

TABLE XIII

COMPARISON OF SUCCESS WITH UNFAMILIAR AND FAMILIAR
VOCABULARY AS MEASURED BY USE OF 15 CRITERIA

	Unfamiliar Vocabulary			Familiar Vocabulary			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	10847	5325	49.09	10957	5565	50.79	1.70
Bright Group	3702	2091	56.48	3742	2167	57.91	1.43

TABLE XIV

UNFAMILIAR AND FAMILIAR VOCABULARY—MULTIPLICATION
COMPARED BY USE OF 15 CRITERIA

	Unfamiliar Vocabulary			Familiar Vocabulary			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5469	3234	59.13	5500	3477	63.22	4.09
Bright	1875	1255	66.93	1886	1349	71.53	4.60

RESULTS WITH THE THORNDIKE CRITERION

The next group of tables shows the effect of using familiar vocabulary when word position in the Thorndike *Word Book* is used as the criterion. Although by no means adequate, it serves the purpose better than the measure previously reported.

TABLE XV

COMPARISON OF SUCCESS WITH UNFAMILIAR AND
FAMILIAR VOCABULARY

(THORNDIKE *Word Book* CRITERION)

	Unfamiliar Vocabulary			Familiar Vocabulary			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	10852	5066	46.68	10952	5824	53.18	6.50
Bright	3703	1997	53.93	3741	2261	60.44	6.51
Average	4374	2009	45.93	4401	2310	52.49	6.56
Dull	2775	1060	38.20	2810	1253	44.59	6.39

TABLE XVI

COMPARISON OF SUCCESS WITH UNFAMILIAR AND
FAMILIAR VOCABULARY

DIVISION PROBLEMS (THORNDIKE CRITERION)

	Unfamiliar Vocabulary			Familiar Vocabulary			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5393	1835	34.03	5442	2344	43.07	9.04
Bright	1830	744	40.66	1853	910	49.11	8.45
Average	2174	732	33.67	2187	928	42.43	8.76
Dull	1389	359	25.85	1402	506	36.09	10.24

TABLE XVII

COMPARISON OF SUCCESS WITH UNFAMILIAR AND
FAMILIAR VOCABULARY

MULTIPLICATION PROBLEMS (THORNDIKE CRITERION)

	Unfamiliar			Familiar			Diff. %
	Att.	R.	%	Att.	R.	%	
Entire Group	5459	3231	59.19	5510	3480	63.16	3.97
Bright	1873	1253	66.90	1888	1351	71.56	4.66
Average	2200	1277	58.05	2214	1382	62.42	4.37
Dull	1386	701	50.58	1408	747	53.05	2.47

Table XV shows consistently a gain of over 6 per cent with familiar vocabulary for all intelligence levels. In division, the bright children gain 8 per cent, the average about 9 per cent, and the dull children over 10 per cent. The gain in division for the entire group is 9 per cent in favor of problems employing familiar vocabulary. In multiplication, the gains are also in favor of familiar vocabulary. The bright group gains about 5 per cent, the average 4 per cent, and the dull children about 2.5 per cent, the entire gain being 4 per cent. It is significant that the greater gains are made in division where the thinking seems most difficult for children. With keener differentiation, such as would be supplied by a suitable criterion developed for use with children's textbooks, it is probable that these differences would be more sharply accentuated. To measure the reliability of the obtained difference for the entire group with Thorndike's criterion, we

have again applied the formula for the reliability of the difference: ¹⁷

$$\sigma \text{ diff.} = .047$$

$$\text{diff.} = .0637 \pm .047 \text{ (In favor of familiar vocabulary)}$$

EFFECT OF ABBREVIATIONS OF UNITS OF MEASURE ON CHILDREN'S WORK IN PROBLEMS

In the introduction to the *Word Book* (p. iv), Dr. Thorndike criticizes the credits allotted abbreviations as being probably too low and less reliable than those for other words. Since only two arithmetics were used in the source material, the criticism is probably not unwarranted. However, because of the frequent occurrence of abbreviations of denominate numbers in arithmetic texts, comparisons have been made to determine whether the abbreviation appears to be an element of difficulty in the 6 B grade. The test material includes 45 problems using units of measure without abbreviation (mile) and 45 problems in which such abbreviations occur (mi.). Table XVIII contrasts percentages of success for each group.

TABLE XVIII

COMPARISON OF CHILDREN'S SUCCESS IN PROBLEMS

	Without Abbreviations	With Abbreviations	
	%	%	Gain %
Entire Group	45.77	46.77	1.00
Bright	52.84	53.42	0.58
Average	45.12	46.15	1.03
Dull	37.39	38.87	1.48

The children do slightly better work when denominate numbers are expressed with the abbreviation. This may be because it is the form to which the textbook has accustomed them. Moreover, teachers are aware of the difficulty that the introduction of the abbreviation presents in Grade IV and hence they do careful and explicit teaching and drill work at this point. The teacher's sensing the difficulty when it occurs goes far toward removing it for children.

¹⁷ Garrett, H. E., *Statistics in Psychology and Education*, p. 129.

SUCCESSFUL PATTERN-FORMS FOR STATEMENT
OF THE ARITHMETIC PROBLEM

With the average group, the form maintaining the best total percentage in the entire test material is the uninteresting, interrogative-complex form employing familiar vocabulary without details. This form holds comparatively good position also with the superior children, as well as with the group as a whole. Sample:

If a curtain cord is $1\frac{1}{8}$ yards long, how many yards of cord will be needed for 27 curtains?

Knight, Ruch, Studebaker, Standard Service Arithmetics, Book V, Scott, Foresman and Co. (Adaptation from problem 14, p. 195.)

There is probably no one best way to formulate the arithmetic problem. In division problems, for instance, the form scoring highest is the interrogative-complex, with unfamiliar vocabulary, details, and interest. One explanation is that the potency of the interrogative-complex sentence in division has probably outweighed the difficulty presented by the other factors. There occurs also a declarative form achieving relative success,—uninteresting, with unfamiliar vocabulary, and without details. In multiplication there is a scattering among successful forms. The bright children, as would be expected, have a range of a half dozen forms in which they do almost equally well. They achieve their highest success with the story form,—the declarative sentence, familiar vocabulary, details, and interest. The average group, on the contrary, does its best work in multiplication with the uninteresting, compressed, declarative form using familiar vocabulary. This range of choices suggests that there may be a variety of forms with which children will achieve relative success. However, the form which here achieved somewhat consistently high total percentages with all groups satisfies the general conditions discovered in that it is interrogative-complex, without details, and in familiar vocabulary.

In the case of pattern-forms to be avoided the evidence is rather clear. In the entire material, with all of the children as a whole, as well as with the bright, average, and dull

groups, the poorest total scores were made with the pattern employing the declarative sentence, unfamiliar vocabulary, details and interest. The next lowest score was made by the companion form in the uninteresting material. Samples:

In competing for the bronze medal given by the Public Athletic League, a boy made a running dash of 65 yds. in $7\frac{3}{5}$ seconds. What was his running rate per second?

In order to enrich the soil in his truck gardens, a farmer plans to use $\frac{7}{8}$ bu. of excellent fertilizer per acre. How many acres will 26 bu. fertilize?

This evidence also satisfies the general findings in that the forms causing greatest difficulty are declarative, with details, and in unfamiliar vocabulary. The two forms achieving the next poorer scores also employ unfamiliar vocabulary. Probably the evidence concerning forms to be avoided is of more practical value than that concerning forms achieving success, in that it guides the textbook maker away from pitfalls while leaving a wide range of forms from which to choose in writing arithmetic problems.

CHAPTER V

CRITICAL ANALYSIS OF CHILDREN'S WORK

A careful analysis of the children's work has been made to detect specific factors which influence pupils in determining the arithmetic operation required. Although unfamiliar vocabulary and details seem definite elements of difficulty, neither they nor the sentence structure explain the wrong reasoning used by the children in certain types of problems. There are other and more potent factors, probably unrecognized by textbook-maker or teacher, which influence children in arithmetic. Table XIX offers percentages of success for the bright group with 120 problems on sixteen pattern-forms. The problems are grouped both for content and for the arithmetical principle involved. The curves of success for the entire material might almost be said to run parallel for bright, average, and dull children.

TABLE XIX

PERCENTAGES OF SUCCESS FOR BRIGHT CHILDREN	%
Long Division.....Money Problems (15).....	91.74
Horizontal Division.....Content varied (41).....	28.22
Travel Problems.....Cancellation Form (16).....	38.93
Money Problems.....Multiplication Fractional Part (14)....	52.58
Multiplication Problems....Integers and Common Fractions (64)	66.26

In the interpretation of the results, a study was made of the operation of the cue as an influence in bright children's work. This analysis yielded evidence of the ways that children think in arithmetic which should prove useful in the improvement of our teaching.

In the case of long division with money as content, the high percentage of success made by the bright pupils (91.7%), the success being relatively high in all groups, is due to the reliability of the cues characteristic of this type of problem and to the ease with which these associations are established. Note the definiteness of typical cues selected from the test material:

What is the price of one yard?
Find the price per bag.
Find the price of each.
What was the cost of one suit?
What must each one give?

"To find the cost of one, divide." This is a cue that works and, in most instances, it may be relied upon without much specific thinking. Previous mention was made of a small group of problems not conforming to the patterns, but of service in analysis of children's mental processes. In one of these the question employs the characteristic "each" with a further conditional clause: "How much will each person's share be if they are all to give the same amount?" *All* suggests multiplication, hence the bright children's score for this problem dropped to 78 per cent. The only other problem in the long division money group which made an average of less than 86 per cent among the bright children was one in which the cue (how much) again connoted multiplication: "How much did he pay for one yard?" With the entire group of children there was evidence of the same erroneous reasoning suggested by these two cues. Children's success appears to be conditioned by the cue. This would seem to be poor mental habit. In travel problems in division, the superior children achieved an average success of about 40 per cent. The successful cues include expressions such as: in one day, average per day, average rate per hour. The cue is again definitely reliable; one divides to find the rate per hour or per day. That the success in this group of problems is not greatly higher is due in some measure to the interference of the multiplication cues "how many" and "how far" and to vocabulary difficulties. The outstanding cause of error is, however, the children's wide failure to recognize or to use the "is contained times" concept in division situations. In horizontal division, the bright group made an average score of but 28.2 per cent, the middle group 22.8 per cent, the slow children 15.3 per cent. Children might have been expected to recognize all of these problems as simple "is contained times" cases in division. Careful scrutiny of the work, however, has detected definite sources of sugges-

tions of wrong reasoning. It seems evident in this study that children react neither to the total situation of the problem nor to the critical facts given, but rather to a phrase, or to even a word, which they have become accustomed to recognize as a cue to a definite operation. They perform the operation, it seems, without testing its application to the given situation.

In the study of vocabulary in arithmetic¹ made by Turner (1924), it was found by a frequency count with a series of four arithmetic texts, that the word carrying highest frequency (726) was "how many," considered as a single word.

In our study of unthinking response to cues, success in division problems asking the question in terms of "how many" has been compared with outcomes in multiplication problems using "how many," percentages of success being given in the following table:

TABLE XX

QUESTION IN TERMS OF *How Many*
PERCENTAGES OF SUCCESS

	Division (25 problems)	Multiplication (31 problems)
Bright Group	25.24%	71.10%
Average Group	19.29%	61.84%
Dull Group	11.59%	53.65%
Entire Group	19.33%	62.93%

Evidently, "how many" is preponderatingly a multiplication cue for 6 B pupils. This is probably the result of too strong welding of this association in children's earlier contacts with arithmetic problems. A typical reaction, by no means an extreme case, is shown by the work of 282 children who attempted the following problem, borrowed almost exactly from the Knight, Ruch, and Studebaker Arithmetic, Book V, page 203:

If tomatoes average 9/16 lb. apiece, how many tomatoes will it take to weigh $36\frac{1}{4}$ lbs? (Experimental group 10.6% success)

¹ Stevenson, P. R., Remedial Arithmetic, Bulletin No. 13, Ohio State University, 1924 (Contains Turner's paper).

In this problem over 125 pupils multiplied and over 35 interchanged divisor and dividend ($9/16 \div 36\frac{1}{4}$), there being, in addition, other wrong choices of operation. The "how many" seemingly influenced many children to select multiplication.

The next problem is patterned after one given in the Strayer-Upton Arithmetic for Middle Grades, p. 116, the making of badges being common content in arithmetics:

A lady ordered $32\frac{1}{4}$ yds. of ribbon. How many $\frac{1}{8}$ yd. badges can she make from it? (Experimental group 31.3% success)

Of 302 children attempting this problem, 120 children multiplied. It would seem unwise teaching to permit children to form associations, such as this between "how many" and multiplication, with such strength in early grades as to preclude actual thinking on the child's part in the sixth grade.

In the case of "how many times," the association is even more definitely a multiplication bond with these children. Two division problems are cited from the test material, the interrogative expression being in each instance "how many times":

1. How many times can a pitcher holding $\frac{4}{5}$ gal. be filled from a tank holding 6 gallons? (Pilot Arithmetic, Book II, p. 103)

2. A mast 7 yards long is how many times as long as a stick $\frac{7}{8}$ of a yard long? (See Pilot Arithmetic, Book VI, p. 104 Adaptation)

Of 229 children in the experimental group who attempted the first problem, 12 used the correct principle (5.2 per cent). In the second problem the success reached only 18.9 per cent. Evidently, "how many times" means multiplication to these children. Certainly such automatic reaction will not develop habits of thinking such as are indicated in Dr. Dewey's analysis of a complete act of thought. The "how many times" and the "is contained times" concepts are important arithmetical modes of thinking. One recalls Dr. Thorndike's comment:

"Mastery of the 'times as' relation is hard to acquire, but it is well worth acquiring, not only because of its strong

intellectual appeal, but also because of its prime importance in the application of arithmetic." ²

There are better ways of bringing children to acquire such mastery than to permit them to become accustomed to making unthinking response to a cue.

In division where the vocabulary was unfamiliar or the computation difficult, there seemed with a number of problems to be a tendency on the part of some children to turn to subtraction or addition processes. However, as contrasted with the seeming inability to extricate the mind from the association of "how many" with multiplication, the work in division was more independent when the question was given in terms of, *What is (or was) the price, cost, rate, average speed*, and so on. In a group of 12 problems of this type, the bright children made an average score of 69.2 per cent, the average children 63.6 per cent, and the dull group 54.6 per cent. Other cues are operating also, but use of "What" as the interrogative expression evidently provides greater freedom from the shackles of habitual associative response such as characterizes the mental habit with "How many." Children may, as Dr. Garry Myers ³ has expressed it, "read the problem for mere enjoyment." It is evident, however, from the analysis of the work in division that they frequently make their arithmetical response neither to the total situation presented in the problem nor to an essential element or fact given in the statement, but to some familiar word or expression accepted or seized upon as a cue. This cue is often associated definitely in the children's thinking with some one arithmetic operation. Such habits fetter children's minds. They would profit by arithmetic experiences planned rather to develop thinking, problems, for instance, which, although they employ the same interrogative expressions, or cues, demand use of first one and then another arithmetic operation.

The following list offers a number of expressions which

² Thorndike, E. L., *Psychology of Arithmetic*, p. 225.

³ Myers, Garry C., *Textbooks in Arithmetic (Review)*, *Elementary School Journal*, October 1927, p. 150.

occurred in the test material, expressions which might have served as data to guide effective thinking. They are found, however, to have given little suggestion of the pertinency of division to the problems in which they occur:

Among how many can we distribute . . .
How many pieces of this length can be cut from . . .
How many boards can they get out of $28\frac{3}{4}$ feet of lumber?
How many towels can be made from . . .
How many tables will 14 cakes supply at this rate?
What number (catchers' mitts) can be obtained from . . .
How many will it take to weigh . . .
How long will $8\frac{1}{2}$ tons last him? (rate given)

Such expressions are useful data for reflective thinking in arithmetic. They do not occur in the Pressey technical vocabulary of arithmetic, but we need a new, psychological study of arithmetic vocabulary to bring teachers to appreciate the rich and varied data of arithmetical thinking. Pressey's list does not include either "contain" or "is contained times," both of which are fundamental concepts in division. Intelligent study of the ways in which children reason will improve the teaching of division. Men have worked through the ages to evolve the principles which govern this mode of thinking. Certainly we can transmit it to children in such way as to liberate their mental power rather than to, as it were, handicap them by persistent practice in that failure to use the essential data of their problems which results from too great dependence upon cues. That we do not analyze children's mental processes is the cause of much of our waste in teaching.

THE FORCE OF CUES IN MULTIPLICATION

The average success of bright children was 66.3 per cent with the group of 64 multiplication problems. For 31 problems in "how many" their average score was 71.1 per cent. With a smaller group of problems in "how much" the average score was 69.6 per cent. Other expressions which elicited multiplication response were:

What is the total length?
Find the entire weight.
What was their combined length?
Find the total distance covered.

All of these expressions have the denotation of "all," hence with the entire group also they proved successful guides to thinking. As was the case in division, the children failed when a reliable critical word was combined with an unfamiliar word. The success, for instance, was high when the direction read simply: *Find the price*. When asked to find the bargain price or the wholesale price, even bright children frequently lost the way in division. In multiplication, "Find the total yield" results in disturbance. Scores are below 75 per cent in the bright group for multiplication problems employing the expressions,—

How much material will be required?
Find the amount sufficient . . .
What amount of this width cloth must be purchased?
What will be the amount of the bill?

Colloquial expressions common among adults did not serve good purpose as problem questions, as for instance in the use of "raise" instead of "collect" with reference to money. In other instances, low scores were evidently the result of unfamiliar vocabulary:

What distance will he descend . . .
How many guns did we capture?
How many were cash contributions?
How much time must he allow his employees?

The best work in multiplication seems to have been accomplished in problems with food content. The poorest work was done with money problems requiring fractional parts, the average score for the bright group with 14 such problems having dropped to 52.6 per cent. The cause is, however, directly traceable to the undue influence of the cue, in this instance a strongly welded division association which obliterates thinking in multiplication. It was found that "each" proved a dependable division cue. "To find the cost of each" one usually divides. This bond between "each" and division explains the failure of even bright pupils with the multiplication money problems. In the case of the following problem, which will serve as an illustration, among 290 children attempting the problem, there were 96 instances

of division, due probably to the suggestion of "each," reinforced by "apiece":

If a roll of linoleum priced at \$2.75 per yd. is made into runners of $\frac{7}{8}$ yds. apiece, what will be the value of each runner?

In the companion declarative problem over 80 per cent of the children employed division:

Some toweling costing 72 cents per yd. is cut into lengths $\frac{3}{4}$ yd. each. Find the cost of the material in each towel.

The Smith-Burdge Arithmetics, Intermediate Book, Ginn and Co., p. 90.

An analysis of the high points of a recent textbook⁴ includes the following statement:

A definite technique of teaching problem solving is adopted. This consists of the recurrent use of 'cues' which suggest to the pupil what operation to use.

This study would indicate that unthinking, stereotyped reactions will not lead to the useful modes of critical thinking and to the habits of independent analysis that children might be acquiring from experiences in arithmetic.

ANOTHER TYPE OF OPERATION CUE

The numbers themselves in certain relationships influence the children, it would seem, as operation cues. In the introduction to cancellation, children are given unbroken columns of examples of the type $\frac{2}{3}$ of 6, $\frac{3}{5} \times 10$, and so on. Evidently, the association between small numbers and proper fractions crystallizes for multiplication. In a group of 14 problems requiring merely the division of a whole number of one or two digits by a simple proper fraction, superior children made 1094 attempts. They indicated the correct operation in but 196 solutions, their percentage of success being 17.9. That of the average children was 12.7 per cent, while the dull group dropped to 8.3 per cent. These problems involved operations such as the following:

$$\begin{array}{r} 6 \div \frac{4}{5} \\ 5 \div \frac{3}{8} \\ 21 \div \frac{7}{8} \\ 12 \div \frac{5}{6} \end{array}$$

⁴ Buckingham-Osburn, Searchlight Arithmetic, Advertisement, Ginn and Co., 1927.

In the following problem, cited as a sample of the group, the bright children made a score of 10 per cent, the average 6.5 per cent, and the slow exactly 5 per cent:

If, for their daily kind deed, some country school boys are plowing $\frac{2}{3}$ of an acre of ground a day for an old farmer, in how many days will they plow his 6 acre farm?

The children did better work at all levels with division problems involving heavier computations with fractions than they did with these simple operations which occur so frequently in ordinary life situations. Throughout the examination of the children's work, we found little evidence of habits either of estimating before proceeding to the computation or of checking the reasonableness of the result. The prevailing habit would seem to be to label the answer. No doubt many children refer to the problem after the completion of the computation to note seriously what was required. There seems evidence that somewhat habitually and perfunctorily the operation performed is the one previously associated, through persistent, unpsychological drill, with a familiar phrase, word, or combination of numbers which serves as a cue.

HABITS OF COMPUTATION

This report will not go into the matter of computation errors further than to say that analysis of over 35,000 problem solutions has yielded cumulative evidence which, if presented to teachers, may prove of service in the improvement of teaching. One might pause to note that the zero is a constant source of error with these sixth grade children. There were, for instance, over 50 cases of each of the following zero errors in the eight experimental classes:

$$\frac{875}{8} = 19\frac{3}{8}$$

$$3/5 \text{ of } 525 = 45$$

In the case of a problem requiring the following operation, $45)\overline{\$1702}$, over 70 per cent of the children failed to add the two customary zeros for cents, there being 34 pupils in the class achieving the best scores in the testing who made this omission.

CORRELATION OF EXPERIMENTAL ARITHMETIC SCORES
WITH READING SCORES

It may be that the reader will not deem significant the coefficients of correlation between the arithmetic problem scores of the experimental testing in May and the Sangren-Woody Reading scores obtained in the city-wide testing the following September. Dr. Noonan ⁵ (1926) found, however, that the summer vacation does not cause any significant changes in ability to read among fifth and sixth grade children. The writer's study ⁶ of 150 children of the 5 B grade (1927) found some improvement over the summer with 63 cases, hence an r of but .61. The Pearson r for 218 pupils of the experimental group who took both tests is $.34 \pm .04$. Table XXI gives coefficients of correlation between scores in certain reading abilities measured by the Sangren-Woody Test and the arithmetic reasoning scores obtained in the May testing:

TABLE XXI

ARITHMETIC REASONING AND SANGREN-WOODY READING TEST

Arithmetic Reasoning and

a. Total Reading Score	$r .340 \pm .040$ (k .941)
b. Word Meaning	$r .313 \pm .041$ (k .949)
c. Factual Material	$r .339 \pm .040$ (k .941)
d. Central Thought	$r .290 \pm .042$ (k .957)
e. Organization	$r .301 \pm .041$ (k .954)
f. Total Meaning	$r .246 \pm .043$ (k .969)

For a group of 100 gifted children, DeVoss ⁷ (1925) reports the following correlation coefficients for arithmetic reasoning and Stanford reading scores:

Arithmetic Reasoning and Total Reading Score	$r .43$
Arithmetic Reasoning and Word Meaning	$r .37$
Arithmetic Reasoning and Sentence Meaning	$r .37$
Arithmetic Reasoning and Paragraph Meaning	$r .36$

These coefficients are not greatly different from those found

⁵ Noonan, Margaret, Influence of the Summer Vacation on the Abilities of Fifth and Sixth Grade Children, Teachers College Contribution to Education, No. 204, 1926, p. 67.

⁶ Kramer, Grace A., Do Children Forget During the Vacation? Baltimore Bulletin of Education, December 1927, p. 56.

⁷ Terman, L. W., Genetic Studies of Genius, Vol. I, p. 319.

in the Baltimore experimental group with a more typical spread of intelligence. Efficient reading *is* thinking, but in dealing with the arithmetic problem children do not use the effective types of interpretation and selective thinking which they are trained to employ with ordinary reading material. Nor do they proceed by the steps given in Dewey's analysis of a complete act of thought. It would seem rather that they are apt to omit the step of actually defining the problem difficulty and to respond without deliberation to a familiar cue which may or may not suggest the operation which applies in the given situation.

CHAPTER VI

SUPPLEMENTARY INVESTIGATION

THE RELATION BETWEEN CHILDREN'S EXPRESSED INTERESTS IN ARITHMETIC PROBLEMS AND THE *A PRIORI* JUDGMENTS OF THE STUDY

The criteria of interest employed in the selection and formulation of the interesting arithmetic problems for this study were secured from previous investigations dealing with the determination of children's reading and activity interests. The supplementary study reported in this chapter throws light on the question as to whether these interests are potent also in arithmetic problems. This second investigation has been planned and carried through to answer the criticism that *a priori* judgments were accepted in this matter of children's interests rather than experimental evidence.

To determine children's preferences, a block of 36 problems, comprising about one-third of the original material, was selected for study, the problems being chosen in three groups. Set A is made up of 12 problems with a difficulty range of from 70 to 95 per cent, as determined by the percentage of "principle right" in the previous experiment. Set B comprises 12 problems within the difficulty interval of 60 to 69 per cent, and Set C 12 problems where the success was in each instance less than 20 per cent. Each set contains six interesting and six uninteresting problems. To give due weight to the other factors of the study,—sentence form, vocabulary, and detail—it was arranged that in the interesting group in each set there should be three problems in the declarative form and three in the complex-interrogative form; three from the unfamiliar and three from the familiar vocabulary group; three with and three without details. The same measures were observed in selecting the uninteresting problems for Sets A, B, and C.

To insure chance sequence in presenting the problems to the children, the problem cards of each set were shuffled, tossed up, and typed in the order in which they were picked

up. In form and appearance the supplementary tests are similar to the tests used in May 1928.

The supplementary tests were given February 27-March 11, 1930, in six 6 B classes, this being the grade used throughout the study. Certain preliminary try-out with another group of classes had served to work over problems incident to securing response and to development of directions. The schools chosen for this phase of the investigation represent varied types of school population, No. 13 being a platoon school.

SCHOOLS SERVING IN THE SECOND EXPERIMENT

- No. 13 Tench Tilghman School—Patterson Park Avenue and McElderry Street
 No. 51 Waverly School—34th and Frisby Streets
 No. 64 Liberty School—Garrison and Maine Avenues
 No. 87 Windsor Hills School—Alto and Mt. Holly Streets
 No. 214 Guilford School—York Road and Cold Spring Lane
 No. 236 Hamilton School—Old Harford Road and Christopher Lane

The intelligence, or academic ability status, of this group as compared with the 115 classes making up the 6 B population of the Baltimore City white schools is as follows, the data for the experimental group being based upon the I.Q. of the individual pupils and that for the city upon class medians as of February 1, 1930:

TABLE XXII

	Median I.Q.	Q.
Experimental Group	105.2	11.5
City-Wide 6 B Population	95.8	6.4
Norm	101	

Expressed in ranges of ability grouping, the children of the six classes fall into the following classification:

TABLE XXIII

X—I.Q. 110 or above—93 pupils
Y—I.Q. 90 to 109 —92 pupils
Z—I.Q. Less than 90—34 pupils

The median age of the group is 11 years, 2 months with Q. 6.5 months, the normal age for 6 B being 11 or 11½ years.

The tests were administered in each instance by the writer at intervals of at least a day. The following directions were given each class on the first day, less explanation being required on succeeding days.

TEST DIRECTIONS

Would you like to help make an arithmetic book for sixth grade? Every example in this new book must be interesting to boys and girls. We want to use six of the twelve examples on these sheets. *Read all twelve examples first.* Put O.K. in the margin beside the six you think most interesting. Choose the examples because of the fact that they are interesting to you. Do not mark more than six, but be sure you mark six. Then we will have votes on the first six interesting examples for the new book. I will not leave these papers with your teacher because I am too anxious to see which examples you say are interesting.

After the voting, the following direction was given, "Interesting because" having been written on the blackboard:

Copy "Interesting because" beside each example you marked O.K. and give a good short reason why you say that example is interesting. You know why you voted for it. Don't write a long composition. (Show where to write)

The final direction concerned the solution:

Now work the six examples you marked O.K. so I will know where other boys and girls need help with them.

The reasons for the choices and the solution were asked for in each test, votes on all three forms being obtained from 219 children, 100 boys and 119 girls.

OUTCOMES

The six problems receiving the highest percentages of votes in each test are deemed "interesting" by the new criterion of children's choices. On the first day, five of the six problems originally classified as interesting were chosen; on the second day, four with a fifth tying for place; on the third day, all six of the interesting problems were preferred by the children. The total vote of the eighteen highest percentages gave place to fifteen problems originally classified as interesting with a sixteenth tying for place, if 50.2 per cent and 49.97 per cent may be considered a tied vote.

Based on the fifteen interesting problems chosen also by the children, there is an agreement of $83\frac{1}{3}$ per cent between the criteria originally used in formulating interesting arithmetic situations and the present criterion of the children's own choices. If the tie for place between an interest-

ing and uninteresting problem be accorded a half vote, the agreement reaches 86.1 per cent.

The following table gives the ranking of the problems in each test in the order of the children's votes, the percentage of the entire group voting for the problem, the percentage of success with each problem, and its classification with reference to interest in the first experiment.

TABLE XXIV
RANKING OF THE PROBLEMS BY THE CHILDREN'S CHOICES

Test A			
Rank	Percentage of Group Voting for the Problem	Percentage of Success in Solution	Original Classification
1	79%	89.6	Interesting
2	67%	88.3	Interesting
3	64%	84.3	Interesting
4½	56%	79.1	Interesting
4½	56%	81.8	Uninteresting
6	52%	78.1	Interesting
7	50%	83.5	Uninteresting
8	42%	76.5	Interesting
9	36%	69.6	Uninteresting
10	35%	87.9	Uninteresting
11	34%	81.7	Uninteresting
12	29%	79.7	Uninteresting

Test B			
Rank	Percentage of Group Voting for the Problem	Percentage of Success	Original Classification
1	74%	74.5	Interesting
2	68%	85.2	Interesting
3	60%	85.2	Interesting
4	57%	81.0	Interesting
5	54%	88.1	Uninteresting
6½	50%	79.2	Uninteresting
6½	50%	89.3	Interesting
8	42%	85.7	Interesting
9	40%	77.7	Uninteresting
10½	38%	48.7	Uninteresting
10½	38%	58.7	Uninteresting
12	29%	60.0	Uninteresting

Test C

Rank	Percentage of Group Voting for the Problem	Percentage of Success	Original Classification
1	75%	14.0	Interesting
2	67%	16.6	Interesting
3	64%	10.8	Interesting
4	63%	20.3	Interesting
5	57%	16.3	Interesting
6	51%	32.7	Interesting
7	49%	27.4	Uninteresting
8	47%	16.7	Uninteresting
9	37%	8.7	Uninteresting
10	32%	24.7	Uninteresting
11	30%	19.7	Uninteresting
12	27%	11.9	Uninteresting

The following section offers the problems arranged in the order of the children's choices for each of the three successive days of voting:

SET A—ARRANGED IN ORDER OF CHILDREN'S PREFERENCES

1. A poor girl has to pay \$24.45 for a glass door she broke when a child pushed her against it. There are 45 children in our class and we have decided to share equally the cost of the door. What must each one give?

2. If we help Tom to buy a used wheel marked \$19.75, so we can all ride together, how much must we earn each day, provided we have 17 days to raise the money?

3. If the gymnasium teacher spent \$133.75 for 29 basket-balls this season, what price basket-balls did he buy?

4½. A boy can do 4/5 of the work on a bird house in a week. How many houses can he build in 7 weeks?

4½. A butcher has 75 customers each of whom buys a 3½ pound roast of beef on Saturday. How many pounds of beef must he have on hand to supply them?

6. If a schoolgirl can prepare a luncheon in 17½ hrs., how much time will be spent in making 7 luncheons?

7. A clerk sold 37 pillows at a bargain sale and took in \$36.26 for them. What was the bargain price of one pillow?

8. A dealer purchased 45 bicycles at a total cost of \$1702. Find the price of each.

9. If a farmer eats 25½ pounds of meat in a week, how much meat will he use in 17 weeks?

10. If the anthracite coal used in heating our church for a period of 120 da. costs \$81.60, what is the cost per day?

11. An importer invested \$277.50 in 37 traveling bags. Find the cost per bag.

12. If a cafeteria uses an average of 57½ lbs. of butter per wk., how much will it use in 25 weeks?

SET B—ARRANGED IN ORDER OF CHILDREN'S PREFERENCES

1. We are after the fox that is killing our chickens. Six boys will take turns to-night watching for him with a rifle, each boy to watch $1\frac{5}{6}$ hours. How many hours will the six of us watch?
2. Schoolgirls require $2\frac{1}{2}$ yds. of sateen to make a pair of gymnasium bloomers. Find the amount of sateen sufficient for 27 pairs.
3. Tom paid \$17.10 for 18 yards of tent canvas. How much did he pay for one yard?
4. Allowing $2\frac{1}{4}$ yds. to a bungalow apron dress, how much material will the class require to cut out 25 of these dresses?
5. A carpenter works $7\frac{1}{4}$ hours a day. How many hours will he work in 47 days?
- 6½. If a package of cotton yarn makes $5\frac{1}{2}$ yds. of gingham, how many yards of gingham will 250 packages make?
- 6½. If a package of rare foreign postage stamps fills $\frac{7}{8}$ of a page in our stamp collection book, how many pages will 9 similar packages fill?
8. If our school recipe for muffins calls for $\frac{3}{8}$ lb. of butter for a dozen muffins, what quantity of butter will be required for 7 doz. muffins for the luncheon we are making for the principal's visitors?
9. A fruit grower raising walnuts for a foreign market finds the average yield of an acre in his grove to be $96\frac{1}{2}$ bu. Find the total yield of 27 acres.
- 10½. Nails sell at \$1.04 a pound. Find the price of a package of nails weighing $\frac{3}{8}$ of a pound.
- 10½. If 705 people work in the Boston Shirt Factory, and $\frac{3}{5}$ of them are men, how many of these people are men?
12. If a merchant tailor allows $3\frac{1}{8}$ yd. of double width material for an overcoat, what amount of this width cloth must he purchase to fill an order for 140 overcoats?

SET C—ARRANGED IN ORDER OF CHILDREN'S PREFERENCES

1. The cooking class is making a party for all the first grade children. We use $\frac{2}{3}$ of a pound of butter for an orange cake. How many cakes can we make with 8 pounds of butter?
2. If $\frac{4}{5}$ of a chocolate cake is sufficient for a picnic table, how many tables will 8 cakes supply?
3. If, for their daily kind deed, some country school boys are plowing $\frac{2}{3}$ of an acre of ground a day for an old farmer, in how many days will they plow his 6 acre farm?
4. If the boys are making a dog house with boards $4\frac{1}{4}$ feet long, how many boards can they get out of $28\frac{3}{4}$ feet of lumber?
5. Our garden club sows $\frac{3}{8}$ of a package of seeds to a row of radishes. How many rows will 14 packages sow at this rate?
6. In competing for the bronze medal given by the Public Athletic League, a boy made a running dash of 65 yds. in $7\frac{3}{5}$ seconds. What was his running rate per second?
7. If a towel is $\frac{7}{8}$ of a yard long how many towels can be made from a piece of goods $17\frac{1}{2}$ yards long?
8. A loaf of bread can be made from $\frac{3}{4}$ of a pound of flour. How many loaves will a 15 pound sack make?

9. In order to enrich the soil in his truck gardens, a farmer plans to use $\frac{7}{8}$ bu. of excellent fertilizer per acre. How many acres will 26 bu. fertilize?

10. Allowing $\frac{5}{8}$ of a yard to a vest for a suit of clothes, how many vests can a men's tailor cut from a roll of cloth containing $55\frac{1}{2}$ yards?

11. If tomatoes average 9/16 lb. apiece, how many tomatoes will it take to weigh $36\frac{1}{4}$ lbs.?

12. A pint of silver skin onions weighs $\frac{4}{5}$ of a pound. How many pints are there in a bag of onions in the cellar weighing 22 pounds?

ANALYSIS OF INTERESTING PROBLEMS FAILING
TO WIN PLACE BY THE CHILDREN'S VOTES

The two interesting problems definitely failing to maintain placement as interesting by the children's votes fell in each instance in eighth place. In Set A the problem was as follows:

A dealer purchased 45 bicycles at a total cost of \$1702. Find the price of each.

That the children are interested in wheels and "bikes," as they more frequently call them, is shown in another problem in this test in which a human interest situation revolving about the purchase of a "used wheel" called forth a large vote. Many children's reasons for voting indicated that they were interested in riding or securing wheels. A factor in the failure of the present problem to engage the interest of a larger percentage of children may be the unfamiliar vocabulary, "A dealer purchased 45 bicycles at a total cost" being adult rather than child phraseology.

In Test B the interesting problem which failed to attain place was again of unfamiliar vocabulary for children, in this case a problem with details:

If our school recipe for muffins calls for $\frac{3}{8}$ lb. of butter for a dozen muffins, what quantity of butter will be required for 7 doz. muffins for the luncheon we are making for the principal's visitors?

Although the problem involves a girl's activity, had the vocabulary been familiar it would probably have intrigued the interest of a number of the boys also because of the school situation presented. The words *recipe*, *muffins*, *quantity*, and *luncheon*, no doubt, caused many children, the boys especially, to fail to evaluate it for interest.

The interesting problem which tied for sixteenth place in the voting was also from the original unfamiliar vocabulary group. It seems to present as content an activity of appealing interest since almost all of the children selecting it gave as reason some variation of "saving stamps." It is not unreasonable, probably, to suppose that some of the children had had little experience with certain of the words occurring in the problem:

If a package of rare foreign postage stamps fills $\frac{7}{8}$ of a page of our stamp collection book, how many pages will 9 similar packages fill?

THE CHILDREN'S REASONS FOR THEIR PREFERENCES

This section gives selections from the reasons given by the children for voting for the problems in the first test. Since the direction was to vote for six of the twelve problems given, when a problem originally classified as interesting was passed over, the vote went to an uninteresting problem. Every problem in the tests, therefore, secured votes with reasons given. In addition to all of the problems of the first test, we have cited from Tests B and C the two uninteresting problems which obtained the highest percentages of votes given by the children to uninteresting problems included in those tests. (Test B, 54 and 50 per cent; Test C, 49 and 47 per cent).

TEST A. IN ORDER OF CHILDREN'S PREFERENCES

1. A poor girl has to pay \$24.45 for a glass door she broke when a child pushed her against it. There are 45 children in our class and we have decided to share equally the cost of the door. What must each one give?

(From children's reasons for selecting.)

Interesting because:

- I like to help others.
- It shows how nice the class was.
- It is about helping a child.
- It is almost like a little story.
- It wasn't the child's fault.

2. If we help Tom to buy a used wheel marked \$19.75, so we can all ride together, how much must we earn each day, provided we have 17 days to raise the money?

Interesting because:

- It tells about helping Tom.
- It is about a bike.
- I like wheels.

I would like to have a wheel myself.

It shows that all of the boys wanted to help Tom.

3. If the gymnasium teacher spent \$133.75 for 29 basket-balls this season, what price basket-balls did he buy?

Interesting because:

I like gymnasium.

I like basket-ball.

It talks about sports.

Basket-ball is good sport.

You could have a lot of fun playing basket-ball.

4. A boy can do $\frac{4}{5}$ of the work on a bird house in a week. How many houses can he build in 7 weeks?

Interesting because:

I like birds.

It is about bird houses.

I like to make bird houses.

It is about woodwork.

Most children like to build.

5. A butcher has 75 customers each of whom buys a $3\frac{1}{2}$ pound roast of beef on Saturday. How many pounds of beef must he have on hand to supply them?

Interesting because:

I like roast beef.

It gives you an idea how much a butcher has to have on hand.

It is good to find out how butchers supply people.

That is what I generally get.

It is multiplication.

6. If a schoolgirl can prepare a luncheon in $1\frac{7}{8}$ hrs., how much time will be spent in making 7 luncheons?

Interesting because:

I would like to do the same thing myself.

I like to read about a girl or boy like myself.

I am a schoolgirl and I like to prepare luncheons.

It is about a schoolgirl and her luncheon and the sixth grades take it.

She can work fast.

7. A clerk sold 37 pillows at a bargain sale and took in \$36.26 for them. What was the bargain price of one pillow?

Interesting because:

It is about a clerk.

It is about a bargain sale.

To see how cheap he sold them.

Without pillows you cannot sleep very comfortable.

I want to see how cheap he sold them.

8. A dealer purchased 45 bicycles at a total cost of \$1702. Find the price of each.

Interesting because:

It is long division.

I would like to buy a wheel.

I am getting a bicycle soon myself.

I have a bicycle.

I like to ride a bicycle.

9. If a farmer eats $2\frac{5}{8}$ pounds of meat in a week, how much meat will he use in 17 weeks?

Interesting because:

I like meat.

It is easy to find out what kind of arithmetic to use.

The farmer eats so much meat.

He has a lot of meat.

I don't think he could eat that much meat.

10. If the anthracite coal used in heating our church for a period of 120 da. costs \$81.60, what is the cost per day?

Interesting because:

It is about a church.

We all go to church.

It tells about heating a church.

It tells about something you studied.

Coal makes the church comfortable.

11. An importer invested \$277.50 in 37 traveling bags. Find the cost per bag.

Interesting because:

It is about traveling.

It tells about traveling bags.

I like things that are invested.

You would like to know how much money a man would trust in a bag.

It tells you how to save.

12. If a cafeteria uses an average of $57\frac{1}{2}$ lbs. of butter per wk., how much will it use in 25 wks.?

Interesting because:

We use butter.

Tells the facts you have to look for.

It makes you think hard.

In connection with our cooking.

We might work in a cafeteria some time.

Test B

(The two uninteresting problems receiving the highest percentages of votes given uninteresting problems in this test.)

5. A carpenter works $7\frac{1}{4}$ hours a day. How many hours will he work in 47 days?

Interesting because:

I like to work with wood.

I like carpentering.

It tells how long a carpenter works.

My father is a carpenter. I am interested in what my father does.

Boys in our class are good carpenters.

$6\frac{1}{2}$. If a package of cotton yarn makes $5\frac{1}{2}$ yds. of gingham, how many yards of gingham will 250 packages make?

Interesting because:

We learn all about cotton.

We are studying cotton.

You have to multiply.

It is for girls and we go to sewing.

Interest also expressed in the question asked by the problem.

Test C

(The two uninteresting problems receiving highest percentage of votes given to uninteresting problems in C.)

7. If a towel is $\frac{7}{8}$ of a yard long, how many towels can be made from a piece of goods $17\frac{1}{2}$ yards long?

Interesting because:

I like to make towels.

It has something to do with yards.

It is about measuring.

I used to take sewing and found it interesting.

It tells how much it takes to make a towel.

8. A loaf of bread can be made from $\frac{3}{4}$ of a pound of flour. How many loaves will a 15 pound sack make?

Interesting because:

I like bread.

Everyone needs bread.

Bread is healthy and good for children.

It tells how much flour is needed for bread.

We might grow up to be a baker.

SUMMARY

This supplementary investigation was undertaken to obtain experimental evidence as to children's actual preferences concerning the arithmetic problems previously classified as interesting and uninteresting by criteria obtained from available studies of children's reading and activity interests. A representative block of problems from the original material has been selected with reference both to difficulty value, as established by the previous investigation, and to the other factors of the major study,—sentence form, vocabulary, and details.

The agreement between the children's choices, as expressed by their votes, and the original classification of the problems as interesting and uninteresting is $83\frac{1}{3}$ per cent, 86.1 per cent if a tied vote be recognized. Unfamiliar vocabulary seems to have been a factor obstructing more perfect agreement.

This outcome tends to support our previous assumption that criteria of children's interests in reading and in varied activities may be applied also to interest as a factor in arithmetic problems.

CHAPTER VII

SUMMARY AND CONCLUSIONS

THE PROBLEM OF THE STUDY

This study was undertaken to investigate the effect of four factors upon sixth grade children's success in solving verbal arithmetic problems. The factors chosen were:

1. Interest
2. Sentence Form
3. Style-Language Details
4. Vocabulary

The questions upon which the study attempts to throw light are as follows:

1. Will children achieve better results in arithmetic with problems dealing with children's interests, their games, home and school activities, than they will with the traditional materials of arithmetic?

2. Does it improve their work to employ the declarative form of statement rather than the more usual complex-interrogative sentence?

3. Are children more successful with arithmetic problems employing language details than they are with problems giving only the concise statement of the essential facts of the problem?

4. Do they make higher scores with problems in which the vocabulary is limited to words probably familiar to sixth grade children?

The research tests were formulated to serve the purposes of the study. They provide for comparisons of success with equal groups of problems presenting the two contrasting phases of each experimental factor. To vary these factors one, two, three, and four at a time required the use of sixteen pattern-forms.

The criteria of interest were secured from previous studies in children's reading and activity interests. To determine whether these interests are potent also in arithmetic, a supplementary study was made in which a representative block of the interesting and uninteresting problems was submitted to children's choices. Their expressed preferences show an

agreement of 83 per cent with the original classification. Better agreement was evidently obstructed by unfamiliar vocabulary.

In the absence of a scale for measuring word usefulness for grammar grade children, the scale position of words in the Thorndike *Word Book* was used to indicate the familiarity or unfamiliarity of the research test vocabulary. The further attempt to grade the problems for vocabulary by a cumulative use of fifteen vocabulary studies proved futile. Most of these investigations rate over and again the few thousand commonest English words.

The coefficient of reliability for the experimental arithmetic tests is $.93 \pm .008$. This coefficient expresses a measure of the confidence which may be placed in the test material as a measuring device. It here indicates a fair degree of consistency in performance on the part of the children.

The difference in achievement in arithmetic reasoning between boys and girls is negligible in these results, but 50.4 per cent of the boys having equalled or surpassed the girls' median score. Sex differences in arithmetic are consistently small, but those which have been found to exist, especially in problem solving, are usually in favor of the boys.

There is low, positive correlation between general intelligence and problem solving ability as measured by these tests, the Pearson r being $.386 \pm .037$. This coefficient would indicate merely that the children of better mental ability tended to do better work in arithmetic reasoning. Buckingham also reports low correlation between general intelligence and arithmetic scores.

The Pearson r between the experimental arithmetic scores and chronological age is low and negative, $-.199 \pm .042$. This coefficient indicates that the children who were over-age for the grade did not excel in problem solving.

Between the Baltimore Course of Study Test in Arithmetic and the research tests the Pearson r is $.598 \pm .028$. The course of study test is probably best classified as a test in computation. Dr. Thorndike¹ has predicted that with

¹ Thorndike, Edward L., *The Psychology of Arithmetic*, The Macmillan Co., p. 299.

1000 fourteen year old pupils the r between problem solving and computation would not be over .60.

The research tests were given in May, 1928. On the reopening of school in the fall, in the city-wide testing, these children were given the Sangren-Woody Reading Test. Recent studies tend to show that reading ability with children of this age varies little over the summer vacation. The Pearson r between the experimental arithmetic tests and the reading test is $.34 \pm .04$. From the evidence of other studies offering coefficients of correlation between problem solving and reading tests which measure the various phases of reading ability, the r to be expected would not exceed .45. Terry² (1922) has discovered, however, that the reading of arithmetic problems involves peculiar complexities not measured by reading tests.

SPECIFIC CONCLUSIONS

The difference in success with the uninteresting and the interesting sections of the test material proved negligible. With the entire group it amounted to .6 of one per cent and with the bright group to 1.6 per cent in favor of interesting problems. The same plan of variation with regard to sentence form, details, and vocabulary was used with both types of problems. The children appear to have succeeded as well with the traditional type of arithmetic problem as they did with carefully selected material employing children's interests.

In sentence form, the differences are again small but the case is for the interrogative-complex form. The gain over the declarative form is .6 of one per cent for the entire group and 1.9 per cent for bright children. For the group of problems introduced by the "inverted clause in if," there is a further gain of 1.4 per cent for the whole group and 1.5 per cent for bright pupils. These differences are significant merely as checks upon the newer tendency to write the arithmetic problem in the declarative form.

² Terry, Paul W., *How Numerals are Read: An Experimental Study of the Reading of Isolated Numerals and Numerals in Arithmetic Problems*, Supplementary Educational Monograph No. 18, University of Chicago, 1922.

In the matter of style, the better percentage was earned by the problems employing brief, concise statement of essential facts. The introduction of relevant but unnecessary details is accompanied by a loss of 1.9 per cent for the entire group and 3 per cent for the average group. No superfluous numbers were given in any of the problems.

With use of the best criterion of familiarity of vocabulary available, the outcomes of this study show the success of the entire group to be 6.5 per cent higher with problems using familiar vocabulary. The gains are also consistently over 6 per cent for the bright, average, and dull groups. In division problems there was a loss of 9 per cent for the entire group and 10 per cent for the dull group with the problems employing unfamiliar vocabulary. Were there an accurate measure of vocabulary familiarity or vocabulary utility for children of sixth grade age, these differences might have proved much wider.

This study indicates that there is probably no one best pattern for the statement of the arithmetic problem. Of the sixteen forms tested, that maintaining the best percentage in the entire test material with the average group of children was the uninteresting, interrogative-complex form without details and in familiar vocabulary. This pattern holds comparatively good position also with the superior children and with the group as a whole. It also satisfies the conditions found most effective with regard to sentence form, style, and vocabulary. Certain other forms proved relatively successful. There is a reasonable range of pattern-forms with which the bright pupils succeeded almost equally well.

In the case of pattern-forms to be avoided in stating arithmetic problems, the evidence seems fairly convincing. In the entire material the children were least successful with problems which employed the declarative sentence together with unfamiliar vocabulary and language details. This proved to be the case with the bright, average, and dull groups with both the interesting and uninteresting problems, the percentage being lower with the interesting material. This finding also satisfies the conditions found favorable with reference to sentence form, vocabulary, and details.

The data of this study show poor control of the "is contained times" concept, which is a fundamental principle in arithmetic. The average success with problems of this type was but 28.2 per cent. Children's success in this case appears to be conditioned by the chance right application of their previous association with some word or phrase in the problem which they seize upon as a cue.

PRACTICAL CONCLUSIONS

The differences found in this study are in most instances small but they would seem to be somewhat significant. The recent trend in arithmetic teaching, as expressed in the textbooks of the last five years, is to make the arithmetic problem interesting, to use the declarative sentence form and language details, in a word, to approach the story form. The outcomes of this study would suggest further careful investigation before the school places too great reliance upon interesting content or attractive style to aid to any great extent in developing reasoning power in arithmetic with children.

Throughout the study it is noticeable that children respond to the cue rather than to the facts and requirements of the problem. They appear to do little intelligent estimating or reflective thinking and almost no verifying of their choice of operation. The habit of analyzing children's modes of thinking would probably lead teachers to discourage them from habitually seeking a cue to the operation required. Not to train children to think in the arithmetic problem would seem to be to fail to develop their practical and intellectual interests in arithmetic itself.

There is genuine need for reliable vocabulary studies to measure both the difficulty and utility of vocabulary for older children. A study in the specific vocabulary of arithmetic would prove useful to teachers in the analysis of children's difficulties with arithmetic problems. Both general and specific studies are needed to insure familiar vocabulary for the arithmetic textbook.

APPENDIX

Arithmetic Tests Formulated for This Investigation

The source is given for problems taken *verbatim* from textbooks or adapted to the various pattern-forms used in this study. The tests include a small group of problems which do not follow these forms. The asterisk indicates problems quoted *verbatim*, or substantially so, from the arithmetic cited, textbooks being mentioned in other instances to acknowledge adaptations of their materials to the purpose of this investigation. The permission of the publishers concerned has been granted for this inclusion of their materials. The figures entered with each problem give the percentage right for the entire group of 237 children.

Name..... School No.

ARITHMETIC TEST I

Scored for Principle Right	Percentage Right
1. * How many times can a pitcher holding $4\frac{4}{5}$ gal. be filled from a tank holding 6 gallons?.....	5.2%
Pilot Arithmetic, Bk. II, 1923, p. 103	
2. In competing for the bronze medal given by the Public Athletic League, a boy made a running dash of 65 yds. in $7\frac{3}{5}$ seconds. What was his running rate per second?	17.3%
3. If by employing extra hands a farmer in the grain region can cut his hay and get it into his granary in 143 hrs., how much time must he allow his employees to get in $\frac{3}{4}$ of the harvest?.....	41.3%
Clapp Drill Bk. V, 1926, p. 97	
4. Schoolgirls require $2\frac{1}{2}$ yds. of sateen to make a pair of gymnasium bloomers. Find the amount of sateen sufficient for 27 pairs.....	68.9%
5. * If each catcher's mitt requires $2\frac{3}{4}$ sq. ft. of horse hide, what number of mitts can be obtained from $16\frac{1}{2}$ sq. ft.?	11.5%
Knight, Ruch, Studebaker, Grade VI, 1927, p. 414	
6. * A group of 69 adults agreed to subscribe the sum of \$293.25 for foreign missions. How much will each person's share be if they are all to give the same amount?.....	66.7%
Searchlight, III, 1927, p. 144	
7. If some blue ribbon costing 48¢ per yd. is cut into $\frac{3}{8}$ yd. lengths for badges for the members of our Outdoors Club to wear to our meetings, what will each child's badge cost?.....	49.6%

8. * A package of cereal weighs $1\frac{7}{8}$ lb. Find the entire weight of 25 such packages..... 66.7%
Searchlight, III, 1927, p. 362
9. If the boys must buy 15 tent poles, each $6\frac{1}{2}$ feet long, how many feet of pole will they pay for?..... 70.1%
10. A new hotel in Boston required 798 electric lights. The men put $\frac{7}{8}$ of them on the first floor. How many lights went on this floor?..... 40.6%
11. * Allowing $\frac{5}{8}$ of a yard to a vest for a suit of clothes, how many vests can a men's tailor cut from a roll of cloth containing $55\frac{1}{2}$ yards?..... 9.8%
Smith-Burdge, 1926, p. 115
12. Tom paid \$17.10 for 18 yards of tent canvas. How much did he pay for one yard?..... 69.3%
13. If a farmer eats $2\frac{5}{8}$ pounds of meat in a week, how much meat will he use in 17 weeks?..... 71.9%
14. A child who loves to work in a garden helps every summer morning in several neighbors' garden at 35¢ per hr. He divides his time by giving $\frac{3}{4}$ hr. a morning to each of them. How much does each lady pay him a day? 46.8%
15. A loaf of bread can be made from $\frac{3}{4}$ of a pound of flour. How many loaves will a 15 pound sack make?..... 7.7%
Arithmetical Essentials, Book II, 1926, p. 306
16. If the Camp Fire Girls in our school are going to spend a week camping out on a shore 75 miles down the river, how many miles must they drive their car in an hour to reach the camp in $3\frac{1}{4}$ hours?..... 16.2%

ARITHMETIC TEST II

1. The boys' club will have a fair to raise money for a moving picture machine for our school. There are 126 girls in the sixth grades and $\frac{3}{7}$ of them are making children's dresses to sell at the fair. How many dresses will we have?..... 59.9%
2. If a curtain cord is $1\frac{1}{8}$ yds. long, how many yards of cord will be needed for 27 curtains?..... 70%
Iroquois, Bk. 6, 1927, p. 53
Standard Service, V, 1927, p. 195
3. Our boys can build a boat in $24\frac{1}{4}$ hours. They are working $\frac{5}{6}$ of an hour a day. How many days will the work take?..... 18.6%
4. If 38 church workers agree to collect \$289.80 for some poor families in the neighborhood, how much money must each worker collect?..... 77.5%
Arithmetical Essentials, II, 1926, p. 161
5. A cheap grade of cotton back velvet can be had for \$3.75 a yard. What will $\frac{3}{8}$ of a yard of this velvet for a coat collar cost?..... 50%
6. Tom works in his strawberry patch $1\frac{1}{6}$ hours each day. How many hours will he spend on his strawberries in 27 days?..... 72.1%

7. A man walked $68\frac{2}{3}$ miles in $8\frac{1}{2}$ days. How far did he walk in one day?..... 30.6%
Standard Service, III, 1927, p. 198
8. If, for their daily kind deed, some country school boys are plowing $\frac{2}{3}$ of an acre of ground a day for an old farmer, in how many days will they plow his 6 acre farm? 7.3%
9. On a voyage of $2927\frac{3}{4}$ mi., an ocean liner outstripped her previous record by making the trip in $93\frac{1}{2}$ hrs. Find her average speed per hour on this voyage..... 25.6%
Strayer-Upton, 1928, p. 168
10. If $\frac{4}{5}$ of a chocolate cake is sufficient for a picnic table, how many tables will 8 cakes supply?..... 10.7%
11. An aviator on a flight of 175 mi., flew $\frac{2}{5}$ of the distance through a fog. How many miles did he travel in the fog? 53.3%
12. If a merchant tailor allows $3\frac{1}{8}$ yd. of double width material for an overcoat, what amount of this width cloth must he purchase to fill an order for 140 overcoats? 64.1%
13. Late in the season French Co. put in stock 45 pairs of ball-bearing roller skates at a total cost of \$144.90 wholesale. What was the wholesale price per pair for these skates?..... 75.6%
14. * If tomatoes average $\frac{9}{16}$ lb. apiece, how many tomatoes will it take to weigh $36\frac{1}{4}$ lbs.?..... 10.6%
Standard Service, Bk. V, p. 203
15. * Some toweling costing 72¢ per yd. is cut into lengths $\frac{3}{4}$ yd. each. Find the cost of the material in each towel.. 52.9%
Smith-Burdge, 1926, p. 90
16. If our school recipe for muffins calls for $\frac{3}{8}$ lb. of butter for a dozen muffins, what quantity of butter will be required for 7 doz. muffins for the luncheon we are making for the principal's visitors?..... 62.9%

ARITHMETIC TEST III

1. * If Mrs. Smith used $9\frac{3}{4}$ yds. of narrow plaid silk to make scarfs $1\frac{5}{8}$ yds. in length for a number of customers, how many scarfs did she make?..... 30.5%
Strayer-Upton, 1928, p. 112
2. A dealer purchased 45 bicycles at a total cost of \$1702. Find the price of each..... 84.6%
3. When baseball suits are retailing at \$41.75 per doz., what will be the amount of the bill for $\frac{3}{4}$ doz.?..... 45.9%
4. A fruit grower raising walnuts for a foreign market finds the average yield of an acre in his grove to be $96\frac{1}{2}$ bu. Find the total yield of 27 acres..... 65.5%
Clapp Drill Bk. V, p. 94
5. If the champion junior swimmer in the athletic meet made a record of $27\frac{1}{2}$ yds. in $22\frac{3}{8}$ seconds, what is his rate per second?..... 33.2%

6. * A recipe for cake calls for $\frac{1}{2}$ lb. of sugar. A $6\frac{1}{2}$ lb. bag of sugar is enough for how many times as much cake? 19.3%
Smith-Burdge, 1926, p. 110
7. * If a package of cotton yarn makes $5\frac{1}{2}$ yds. of gingham, how many yards of gingham will 250 packages make? 62.8%
Strayer-Upton, 1928, p. 133
8. The Woodberry School won the district basket-ball championship. The team practiced a total of 103 hrs., $\frac{7}{8}$ of this practice occurring after school. How many hours did they practice after school?..... 51.8%
9. * How much must Mrs. King pay at the store for some fish weighing $\frac{7}{8}$ of a pound and costing 75 cents a pound? 54.1%
Strayer-Upton, 1928, p. 88
10. Our school garden needs 45 poles, each pole to be $5\frac{3}{4}$ feet tall. How many feet of lumber will we use?..... 68.9%
11. * A mast 7 yards long is how many times as long as a stick $\frac{7}{8}$ of a yard long?..... 18.9%
Pilot, II, 1923, p. 103
12. We are going to Washington on Saturday on our wheels to see some strange, fierce animals. The Park is about 44 miles away and we want to reach it in $3\frac{1}{2}$ hours. How far must we ride each hour?..... 32.9%
13. If Tom and his sister plant $3\frac{3}{8}$ rows of beans in their vegetable garden with a large package of beans, how many rows can they plant with 15 of these packages?.. 63.2%
14. If Mr. Jones had 172 pounds of sugar and sold $\frac{3}{4}$ of it, how many pounds did he sell?..... 50.9%
Smith-Burdge, 1926, p. 96
15. If the boys are making a dog house with boards $4\frac{1}{4}$ feet long, how many boards can they get out of $28\frac{3}{4}$ feet of lumber?..... 16.3%
16. A clerk sold 37 pillows at a bargain sale and took in \$36.26 for them. What was the bargain price of one pillow? 79.9%

ARITHMETIC TEST IV

1. Children's running shoes are sold to schools for \$18.25 a dozen pairs. What must we pay for $\frac{3}{4}$ of a dozen pairs? 46.7%
2. If English tea is sent to this country in packages containing $4\frac{1}{2}$ pounds, how many pounds of tea will there be in a large case holding 288 packages?..... 69.2%
Smith-Burdge, 1926, p. 233
3. A western freight train loaded with beef reaches a small town $475\frac{1}{2}$ miles away in $23\frac{3}{4}$ hours. How far does this train travel in an hour?..... 32.3%
Clapp Drill Book in Arithmetic, Book VI, p. 105
4. If we practice jumping and racing games for $\frac{5}{6}$ of an hour a day, how many days will it take us to practice 12 hours?..... 8.2%

5. A carpenter works $7\frac{1}{4}$ hours a day. How many hours will he work in 47 days?..... 69.8%
6. If there are 75 girls in the sixth grades in our school and $\frac{3}{8}$ of them make cakes for our supper in the woods, how many cakes will we have for the supper?..... 55.4%
7. A poor girl has to pay \$24.45 for a glass door she broke when a child pushed her against it. There are 45 children in our class and we have decided to share equally the cost of the door. What must each one give?..... 84.6%
8. * If a towel is $\frac{7}{8}$ of a yard long, how many towels can be made from a piece of goods $17\frac{1}{2}$ yards long?..... 17.4%
Essentials, II, 1926, p. 89
9. Mother is reading $\frac{7}{8}$ of a chapter of Treasure Island to us each night. In how many nights will she finish the 21 chapters?..... 16.7%
10. * If the anthracite coal used in heating our church for a period of 120 da. costs \$81.60, what is the cost per day? Smith-Burdge, 1926, p. 193 80.4%
11. Our class of 47 children is painting a frieze to show the different kinds of ships men have made. Each child's ship picture is $1\frac{1}{4}$ ft. long. How long a border can we make for our walls?..... 59.3%
12. If a roll of linoleum priced at \$2.75 per yd. is made into runners of $\frac{7}{8}$ yds. apiece, what will be the value of each runner?..... 45.2%
13. * In testing a motorcycle the machine was driven 17 $\frac{3}{5}$ mi. in $14\frac{2}{5}$ min. What was the rate of speed?..... 29%
Knight, Ruch, Studebaker, Bk. 6, 1927, p. 97
14. If a package of rare foreign postage stamps fills $\frac{7}{8}$ of a page in our stamp collection book, how many pages will 9 similar packages fill?..... 67%
15. A merchant had a surplus stock of 231 overcoats. At a special reduction sale he disposed of $\frac{3}{7}$ of them. How many overcoats were sold at the bargain price?..... 53.4%
Smith-Burdge, 1926, p. 296
16. Allowing $2\frac{1}{4}$ yds. to a bungalow apron dress, how much material will the class require to cut out 25 of these dresses? 64.1%

ARITHMETIC TEST V

1. If it costs a sporting goods dealer \$215.25 to make 45 Boy Scout uniforms, what is the manufacturing cost of each?..... 91.4%
Iroquois, Bk. 6, 1927, p. 79
2. In order to enrich the soil in his truck gardens, a farmer plans to use $\frac{7}{8}$ bu. of excellent fertilizer per acre. How many acres will 26 bu. fertilize?..... 9.1%
Pilot, Bk. II, p. 174
3. If in a single library period a child reads $1\frac{3}{4}$ chapters of an interesting pioneer history story, how many chapters can he complete in 15 library periods?..... 68%

4. A team secured 615 contributions for a hospital drive. Of these gifts, $\frac{4}{5}$ were in cash. How many were cash contributions? 56.7%
Pilot, II, p. 235
5. At what price is a manufacturer selling radios if he received a total of \$1855 for 35 sets?..... 83.2%
Iroquois, Bk. V, 1926, p. 40
6. * As decorations for our class music festival, we will make sunflowers of pieces of crepe paper $\frac{2}{3}$ yd. long. How many pieces of this length will $6\frac{1}{2}$ yds. provide for?.. 13%
Ruch, Studebaker, Knight, V, 1927, p. 220
7. In measuring ingredients for jam, a housewife allows $\frac{3}{4}$ lb. of sugar to a quart of gooseberries. How much sugar must she allow for 25 qts. of gooseberries?..... 68%
Arithmetical Essentials, Bk. II, 1926, p. 283.
8. When tomatoes are selling for \$1.55 per bu., what must school gardeners ask for $\frac{5}{8}$ bu. baskets of tomatoes?.. 50%
9. * When you have finished $\frac{3}{8}$ of an automobile trip of 330 miles, how far have you gone?..... 58.1%
Smith-Burdge, 1926, p. 290
10. We are after the fox that is killing our chickens. Six boys will take turns to-night watching for him with a rifle, each boy to watch $1\frac{5}{6}$ hours. How many hours will the six of us watch?..... 68.1%
11. If we help Tom to buy a used wheel marked \$19.75, so we can all ride together, how much must we earn each day, provided we have 17 days to raise the money? 83.3%
12. * If Mr. White uses $\frac{2}{3}$ of a ton of coal a week, how long will $6\frac{1}{2}$ tons last him?..... 12.8%
Ruch, Studebaker, Knight, Book III, p. 341
13. If our school band is buying an \$11.75 drum for $\frac{4}{5}$ of the regular price, what are we paying for the drum?.. 52%
Searchlight, III, 1927, p. 16.
14. A butcher has 75 customers each of whom buys a $3\frac{1}{2}$ pound roast of beef on Saturday. How many pounds of beef must he have on hand to supply them?..... 70.6%
Searchlight, III, 1927, p. 289
15. How many times in an afternoon can a young clerk weigh into small boxes $\frac{3}{8}$ of a pound of spice from a 5 pound sack?..... 14.7%
Smith-Burdge, 1926, p. 69.
16. Jack rode $55\frac{2}{3}$ miles in $3\frac{1}{2}$ hours on his wheel. How far did he ride in one hour?..... 37.9%

ARITHMETIC TEST VI

1. * The new style street car that the car company is building this year is $25\frac{1}{2}$ feet long. What is the total length of 36 of these new cars?..... 77.1%
Social Arithmetic, I, 1926, p. 250
2. If an air crew that fell 425 feet from shore floated $\frac{2}{3}$ of that distance, how far did they float?..... 55%

3. Rolls of silk containing 57 yards cost \$128.25. What is the price of one yard of silk?..... 91.7%
Pilot, II, p. 182
4. If on our trip to collect flowers for our books we treat each child to $\frac{3}{8}$ of a pint of ice cream, how many children can we treat with $15\frac{1}{2}$ pints?..... 21.8%
5. Father will buy us a pony to ride if we earn money to feed him. He will eat $\frac{7}{8}$ of a peck of oats a week. How many pecks must we buy in 18 weeks?..... 71.6%
6. At \$1.25 a yard for curtain goods, what will a curtain $\frac{7}{8}$ of a yard long cost?..... 53.1%
Strayer-Upton, Middle Grades, 1928, p. 212
7. A boy can do $\frac{4}{5}$ of the work on a bird house in a week. How many houses can he build in 7 weeks?..... 70.8%
8. If an airplane crossing the New England States flies a distance of $1510\frac{1}{4}$ miles in $17\frac{1}{2}$ hours, how many miles does it travel in an hour?..... 38.6%
9. An Eskimo hunter, on a polar bear hunt, provides $\frac{7}{8}$ lb. of reindeer meat a day for each dog in his team. Among how many dogs can he distribute 21 lb. of meat a day?..... 21.8%
10. * How many miles will a salesman travel on the average per hour if he travels $38\frac{3}{4}$ mi. in $8\frac{1}{4}$ hrs.?..... 33.9%
Standard Service, Grade V, 1927, p. 203
11. The average speed of a certain express train is $87\frac{3}{4}$ mi. per hour. Find the total distance covered in 92 hours.. 67.9%
12. If an Arab and his camel, traveling 231 mi. across a burning desert, were without water $\frac{3}{5}$ of the journey, for how many miles did they travel without water?... 54.8%
13. * A single grapefruit sometimes weighs $\frac{7}{8}$ lb. How many grapefruit of this size would you expect to get in a box of grapefruit weighing $25\frac{1}{2}$ lbs.?..... 14.9%
Searchlight, III, 1927, p. 296 (Test slight misprint)
14. * If a war carrier pigeon flew $1010\frac{1}{2}$ mi. in $57\frac{3}{4}$ hrs., what was his average speed per hr.?..... 37.6%
Smith-Burdge, 1926, p. 233
15. A number of children are catching trout at the average rate of $1\frac{2}{3}$ doz. per hour. How many dozen trout will they catch in 15 hrs.?..... 65.9%
16. If the druggist purchases quinine in large quantities at the wholesale house at \$6.75 per lb., what is the value of his $\frac{3}{8}$ lb. packages?..... 55.3%

ARITHMETIC TEST VII

1. If the gymnasium teacher spent \$133.75 for 20 basket-balls this season, what price basket-balls did he buy?.. 91%
2. A lady ordered $32\frac{1}{4}$ yds. of ribbon. How many $\frac{1}{8}$ yd. badges can she make from it?..... 31.3%
Strayer-Upton, 1928, p. 116
3. * If a cafeteria uses an average of $57\frac{1}{2}$ lbs. of butter per wk., how much will it use in 25 wks.?..... 71.9%
Smith-Burdge, 1926, p. 204

4. Our class is going on a picnic. The girls will bring the lunch and the boys will pay the bus bill of \$7.65, each boy's share to be $\frac{2}{15}$ of this amount. What must Tom pay? 63.8%
5. If it requires an average of $\frac{3}{8}$ of a ton of alfalfa hay to feed one milch cow during a winter month, how many cows in his herd can a farmer feed with 6 tons of hay? Clapp Drill Book V, p. 85 21.9%
6. Colonel Lindbergh achieved an ocean flight of $3027\frac{3}{4}$ mi. in $34\frac{1}{2}$ hrs. What was his average rate per hour? 38.2%
7. In a library card index of 525 volumes, $\frac{3}{5}$ are tales of adventure. How many books of this character are there? 68.8%
8. *For a banquet in a Rochester hotel 14 tables, each $4\frac{2}{3}$ ft. long, were placed end to end. What was their combined length?..... 69.1%
Pilot, II, 1923, p. 192
9. If the girls use $\frac{7}{8}$ of a cup of sugar for a cherry pie, how many cups will they use for 9 pies for the lunch for our school ball teams?..... 70.3%
10. Nails sell at \$1.04 a pound. Find the price of a package of nails weighing $\frac{3}{8}$ of a pound?..... 61.8%
11. If Mary rides $22\frac{1}{2}$ miles on her wheel in $1\frac{1}{4}$ hours, how far can she ride in an hour?..... 34.4%
12. A pint of silver skin onions weighs $\frac{4}{5}$ of a pound. How many pints are there in a bag of onions in the celiar weighing 22 pounds?..... 12.8%
Pilot, II, p. 294
13. If 705 people work in the Boston Shirt Factory, and $\frac{3}{5}$ of them are men, how many of these people are men? Pilot, II, p. 96 61.3%
14. Mother allows us to play jacks or marbles $1\frac{1}{8}$ hours every day. How many hours can we spend playing these games in $21\frac{1}{2}$ days?..... 52.4%
15. If a merchant bought 38 brooms for \$24.70, what was the price of each broom?..... 87.4%
16. We want the prize for our school garden. Our class will give $24\frac{1}{2}$ hours of work to planting seeds this week. How many children must help, if each child works $\frac{3}{4}$ of an hour?..... 28.8%

ARITHMETIC TEST VIII

1. If Tom's father lets school teams have the finest bats for \$7.05 a dozen, how much must the boys in our school raise to get $\frac{2}{3}$ of a dozen at this price?..... 62.3%
2. If it takes $\frac{3}{4}$ of a pound of sugar to preserve a box of a certain fruit, how many pounds of sugar will preserve 15 boxes?..... 66.8%
3. The cooking class is making a party for all the first grade children. We use $\frac{2}{3}$ of a pound of butter for an orange cake. How many cakes can we make with 8 pounds of butter?..... 16.7%

4. If you ride $237\frac{1}{2}$ miles in $3\frac{3}{4}$ hours in a railroad train, how many miles do you go in one hour?..... 42.3%
5. The enemy began a battle with 245 machine guns but the Americans captured $\frac{5}{7}$ of them. How many guns did we capture?..... 61.8%
6. If a man paints $9\frac{3}{4}$ square feet of wall in a new building in the city in an hour, how many square feet can he paint in 45 hours?..... 67.5%
7. A worker in a tailor shop is cutting pieces of cloth $3\frac{5}{8}$ yards long from a new bolt of cloth containing 25 yards. How many pieces of this length can he cut?.... 21.6%
Smith-Burdge, 1926, p. 206
8. If 75 children's swimming suits cost us \$93.75, what was the cost of one suit?..... 91.3%
9. * Among how many children at Tom's surprise party can we distribute $3\frac{7}{8}$ lbs. of homemade chocolate fudge, if we allow $\frac{3}{8}$ lb. to each child?..... 35.3%
Essentials, II, 1926, p. 89
10. Isinglass comes in sheets at \$5.35 per yd. What will be the price of $\frac{3}{8}$ yd. strips of isinglass?..... 59%
11. If a schoolgirl can prepare a luncheon in $1\frac{7}{8}$ hrs., how much time will be spent in making 7 luncheons?..... 71.7%
12. Our garden club sows $\frac{3}{8}$ of a package of seeds to a row of radishes. How many rows will 14 packages sow at this rate?..... 14.8%
13. * If a government inspector, planning an automobile trip of $315\frac{1}{2}$ mi., hopes to reach his destination in $4\frac{1}{2}$ days, how many miles must he average per day?..... 37.6%
Pilot, II, p. 220
14. An importer invested \$277.50 in 37 traveling bags. Find the cost per bag..... 89.3%
Searchlight, III, 1927, p. 481
15. If an Indian shoots the Niagara Falls rapids in his canoe at the rate of $17\frac{1}{2}$ ft. per second, what distance will he descend in $9\frac{1}{2}$ seconds?..... 49.8%
16. * If in an audience of 128 visitors $\frac{5}{8}$ were women, how many women were present?..... 63.7%
Pilot Arithmetic, II, p. 96

DIRECTIONS FOR THE ARITHMETIC TESTS

To be given each day for eight days at nine o'clock beginning with May 3, 1928.

To be read to the children

Here are sixteen arithmetic examples. You must try every example.

Do not use scrap paper or anything else to work on. Put all your work in the big space alongside of the example. You must not write the answer and leave out the work because your work will count. We will give you time to finish, but do not waste time.

Be sure to turn over and do the examples on the other side of the paper.

If you finish before the other children, put your paper safely away in your desk and get out something to read. Do not come up the aisle with your paper.

READY!—Go!

APPENDIX

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